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The differential flow near by polar cap surface of neutron star in the case of inclined magnetic field

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Abstract. The flow due to the closure of the magnetospheric current in a liquid layer at the neutron star surface is considered. The only case of a homogeneous magnetic field is considered, but the possibility of inclination of the field to the neutron star surface is taken into account. It is shown that the inclination of the magnetic field may lead to the appearance of a vertical liquid flow velocity comparable to the horizontal one.

Keywords: pulsars: general; stars: neutron

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1 Introduction

The current braking is one of the main mechanisms of pulsar braking (Beskin 2018). It occurs due to the electric current closure inside the neutron star (Beskin 2018; Sob'yanin 2023). Current distribution in this case has been considered, for example, by Sob'yanin (2024). In the paper, we assume that neutron star crust is covered by a plane liquid layer and consider a liquid flow in the layer caused by current closure.

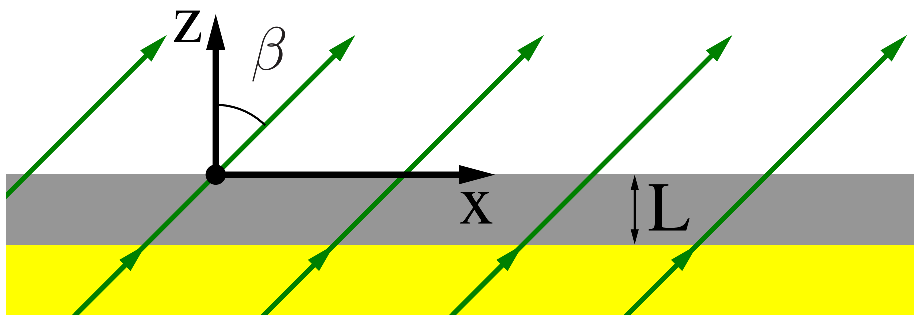

Fig. 1. This sketch shows the liquid layer on the neutron star surface. The liquid layer is shown in gray color, the rigid star crust is shown in yellow color, and the magnetic field direction is shown by green lines.

2 Model

In this paper, we use the same model as Vorontsov & Barsukov 2019; Tsygan et al. 2014. We consider only the simplest case with plane geometry and a homogeneous magnetic field, see Fig. 1, with isotropic conductivity and viscosity. We also use the simplest equation of state $p = p(\rho)$, where p and ρ is liquid pressure and density, respectively, and for the sake of simplicity, we assume that the crust has infinite conductivity. Following Tsygan et al. 2014, we consider only stationary flow. Hence, the magnetic hydrodynamic equations in the frame of references rotating with the neutron star may be written as

$$\rho\,\left(2\,[\mathbf{\Omega}\times\mathbf{v}] + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = -\nabla p + \frac{1}{c}\,[\mathbf{j}\times\mathbf{B}] + \boldsymbol{\mathcal{F}}_{\rm vis} + \rho\,\mathbf{g}, \tag{1}$$

$$-\nabla\varPhi + \frac{1}{c}\,[\mathbf{v}\times\mathbf{B}] = R\,\mathbf{j}\,,\ \ {\rm div}\mathbf{B} = 0\,,\ \ {\rm rot}\mathbf{B} = \frac{4\pi}{c}\,\mathbf{j}\,,\ \ {\rm div}\,(\rho\,\mathbf{v}) = 0, \tag{2}$$

where $\mathbf{v}$ is liquid velocity, $\mathbf{B}$ is magnetic induction, $\mathbf{j}$ is electric current density, $\varPhi$ is electrostatic potential, R is liquid resistivity, $\mathbf{\Omega}$ is angular velocity of the neutron star rotation, $\Omega = 2\pi/P$ where P is radiopulsar period, $\mathcal{F}_{\rm vis}$ is viscous force and $\mathbf{g} = -g\mathbf{e}_z$ is gravitational force and we assume that g is constant. At first, we consider the 0th approximation when magnetosphere electric current is absent and, in this case, we assume that electrical current is the liquid layer and liquid flows are absent, so we have $\mathbf{v}_{(0)} = 0$, $\mathbf{j}_{(0)} = 0$ and also $\nabla p_{(0)} = \rho_{(0)} \cdot g\,\mathbf{e}_z$. We denote the values in 0th approximation by sign $_{(0)}$. In the paper we assume that $\mathbf{B}_{(0)} = B_{(0)}\,(\cos\beta\,\mathbf{e}_z + \sin\beta\,\mathbf{e}_x)$, where $B_{(0)}$ and β are some constants, see Fig. 1. We also assume that $\Phi_{(0)} = 0$ and density $\rho_{(0)}$ depend only on z. Now we consider small perturbations caused by magnetospheric electric current. In the linear approximation of the perturbations we have

$$2\,\rho_{(0)} \cdot [\boldsymbol{\Omega} \times \mathbf{v}] = \frac{B_{(0)}}{c} \cdot [\mathbf{j} \times \mathbf{e}_B] + \boldsymbol{\mathcal{F}}_{\rm vis} - \nabla\delta p - \delta\rho \cdot g\,\mathbf{e}_z, \qquad (3)$$

$$-\nabla\,\Phi + \frac{B_{(0)}}{c} \cdot [\mathbf{v} \times \mathbf{e}_B] = R_{(0)}\,\mathbf{j}\,, \quad {\rm div}\mathbf{j} = 0 \quad \text{and} \quad {\rm div}\left(\rho_{(0)}\,\mathbf{v}\right) = 0, \qquad (4)$$

where $\mathbf{v}$ and $\mathbf{j}$ are considered as perturbations of liquid velocity and electric current correspondingly, $\delta p = p - p_{(0)}$ and $\delta\rho = \rho - \rho_{(0)}$ are perturbations of pressure and density correspondingly, $\mathbf{e}_B = \mathbf{B}_{(0)}/B_{(0)}$ and $c_s^2 = \frac{\partial p}{\partial \rho}(\rho_{(0)})$. Let us consider the boundary conditions $\frac{\partial v_x}{\partial z}\big|_{z=0} = 0$, $\frac{\partial v_y}{\partial z}\Big|_{z=0} = 0$, $j_z|_{z=0} = \frac{1}{\cos\beta} \cdot \hat{j}_B^{\,(m)}\Big|_{z=0}$ and $\Phi|_{z=-L} = 0$, $\mathbf{v}|_{z=-L} = 0$, where $\mathbf{j}^{\,(m)} = j_B^{(m)}\,\mathbf{e}_B$ is electric current flow in magnetosphere. In the case of ${\rm Ha}^2 \gg E^{-1} \gg 1$, where ${\rm Ha} = (B_{(0)}L)/(c\sqrt{\eta_{(0)}R_{(0)}})$ is Hartmann number, $E = \eta_{(0)}/(\Omega L^2 \rho_{(0)})$ is Ekman number and $\eta_{(0)}$ is liquid shear viscosity coefficient, we can use "force free" approximation outside the boundary layers. Hence the approximate solution of equations (3) and (4) outside the boundary layers may be written as

$$v_x = -\frac{1}{\cos^2\beta} \cdot \frac{c^2}{B_{(0)}^2} \cdot (K_x - K_x') \quad \text{and} \quad v_y = \frac{1}{\cos^2\beta} \cdot \frac{c^2}{B_{(0)}^2} \cdot \left(K_y + K_y'\right), \qquad (5)$$

$$v_z = {\rm tg}\beta \cdot \frac{c}{B_{(0)}} \cdot \frac{1}{\rho_{(0)}(z)} \cdot \frac{\partial \hat{j}_B}{\partial y} \cdot \tilde{K}_z(z), \qquad (6)$$

where we define $K_x = R_f\,\frac{\partial\delta\hat{p}_0}{\partial\tilde{x}} + {\rm tg}\beta\,\tilde{R}_f\,\frac{\partial^2\delta\hat{p}_0}{\partial y^2}$, $K_x' = \frac{B_{(0)}}{c} \cdot \frac{\partial \hat{j}_B}{\partial y} \cdot \left(\tilde{R}_{(0)} + \frac{\sin^2\beta}{\rho_{(0)}(z)}\,\tilde{K}_z(z)\right)$ and $K_y = -R_f\,\frac{\partial\delta\hat{p}_0}{\partial y} + {\rm tg}\beta\,\tilde{R}_f\,\frac{\partial^2\delta\hat{p}_0}{\partial\tilde{x}\,\partial y}$, $K_y' = \frac{B_{(0)}}{c} \cdot \left(\sin\beta\cos\beta\,R_{(0)}\,\hat{j}_B - \tilde{R}_{(0)}\,\frac{\partial\hat{j}_B}{\partial\tilde{x}}\right)$ and also we introduce $\tilde{K}_z(z) = \int_{-L}^{z} \rho_{(0)}(z')\,R_{(0)}(z') \cdot (f(z')\,K_0 - 1)\;dz'$, $\tilde{R}_{(0)}(z) = \int_{-L}^{z} R_{(0)}(z')\,dz'$, $R_f(z) = R_{(0)}(z) \cdot f(z)$, $\tilde{R}_f(z) = \int_{-L}^{z} R_f(z')\,dz'$ and the value K_0 is defined as $K_0 = \int_{-L}^{0} \rho_{(0)}(z')\,R_{(0)}(z')\,dz' \,/\, \int_{-L}^{0} \rho_{(0)}(z')\,R_{(0)}(z')\,f(z')\,dz'$. The function $f(z)$ is defined as

$f(z) = \exp\left(\int_z^0 g \,/\, c_s^2(z')\, dz'\right)$. The values $\delta\hat{p}_0$ and $\hat{j}_B$ depend only on $\tilde{x} = x - \mathrm{tg}\beta\, z$ and y. And the function $\delta\hat{p}_0$ may be written as

$$\delta\hat{p}_0(\tilde{x}, y) = \cos\beta \cdot \sin\beta \cdot \frac{B_{(0)}}{c} \cdot \frac{K_0}{2\pi} \cdot \int_{-\infty}^{+\infty} \ln(\tilde{r}) \cdot \frac{\partial \hat{j}_B}{\partial y}(\tilde{x}', y')\, d\tilde{x}'\, dy', \qquad (7)$$

where $\tilde{r} = \sqrt{(\tilde{x} - \tilde{x}')^2 + (y - y')^2}$. The pressure and density perturbations δp and $\delta\rho$ are equal to $\delta p = \delta\hat{p}_0(\tilde{x}, y) \cdot f(z)$ and $\delta\rho = \delta p / c_s^2(z)$ correspondingly. The function $\hat{j}_B(\tilde{x}, y)$ is defined as $\hat{j}_B(\tilde{x}, y) = \hat{j}_B^{(m)}(x = \tilde{x}, y, z = 0)$.

3 Results

The solutions (5) and (6) show that vertical flow velocity may sometimes be comparable with horizontal flow velocities. For, example, in the case of $\rho \sim 10^6\,\mathrm{g\,cm^{-3}}$ (Haensel et al. 2007), $\eta_{(0)} \sim 10^4\,\mathrm{g\,cm^{-1}\,s^{-1}}$ (Chugunov & Yakovlev 2005; Ofengeim & Yakovlev 2015), $R_{(0)} \sim 10^{-19}$ SGS (Potekhin 1999) we have $E \sim 10^{-11}$ and $\mathrm{Ha} \sim 10^{11}$. So in the case of $\beta \sim \frac{\pi}{4}$, $P \sim 1$ s, $B_{(0)} \sim 10^{12}$ G and $j_B^{(m)} \sim \Omega B_{(0)}/(2\pi c)$ we may estimate $v_z \sim v_x \sim 10^{-10} - 10^{-8}\mathrm{cm\,s^{-1}}$. It means that electric current almost does not close in the liquid layer, which is consistent with the result of Sob'yanin (2023) and supports the conclusion of Sob'yanin (2024) that pulsar J0901-4046 brakes due to electric closure in the rigid crust or possibly in deeper layers. It is also possible that such flow may lead to a very slowly growing instability similar to the one that was considered in Kuznetsov & Mikhailov 2024.

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