



## Formation of single stars and their planetary systems, typicality of the Solar system

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**Abstract.** In this paper, we determine the dynamic viscosity for the case of pulsating disturbances in the density of gas-dust disks. The characteristic velocities of the disk dust particles are divided into separate classes of orbits with Keplerian and near-Keplerian motions. For these classes, we obtain formulas for the dynamic viscosity. We analyze the distributions of exoplanets by their masses, major semi-axes, and eccentricities of their orbits. We conclude that hot giant planets can be located on complex, unstable orbits of accretion onto the central star.

**Keywords:** protoplanetary cloud; gas-dust disk; dynamic viscosity

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## 1 Introduction

In 1980, as a result of the efforts of many researchers, a standard model of the origin of the Solar System was built. The main problems that are not solved in the standard model were discussed in 2010 at a workshop at the SAI MSU (Vityazev & Pechernikova 2010). Two of the most important problems were identified: 1) to date, the correspondence between the stages of evolution of a protoplanetary cloud and the stages of evolution of the Sun at the protostar stage has not been determined; 2) the growth mechanisms of planetesimals, ranging from the size of dust grains ( $10^{-5}$  cm) to ten-kilometer sizes, have not yet been clarified. More recent results of research into the problem of the formation and evolution of planetary systems are discussed in monographs of Safronov 1969 and Armitage 2010. The monograph lists the main problems, the solution of which can help explain the features of the large-scale structure of the Solar System: 1) How were the Terrestrial planets and giant planets formed? 2) What mechanisms determined the final structure of the Solar system? 3) What are the reasons for the deficiency of angular momentum of the Sun? 4) If we draw an analogy with the scale of electromagnetic waves, then what is the modern scale of gravitational waves? According to Hawking and Werner, the frequency of gravitational waves can vary from extremely low to high frequencies. The gas and dust disks of the stars MWC 758, HD 100453, HD 135344B have a spiral structure. Large-scale structures of such disks can become the basis for constructing a model of the formation of planetary systems and for testing the adequacy of the model. The dynamic viscosity of motion in gas-dust disks is obtained from the general laws of motion (Navier–Stokes equation) by analytical methods of calculations under certain restrictions on the velocity field.

## 2 Dynamic viscosity of $K_w$ and $K_r$ disks

We will consider two main mechanisms for the accumulation of dust particles in gas-dust disks. One of them is the inelastic collision and adhesion of dust (the problem of three bodies with two “zero” masses, Abdulmyanov 2011). According to this model, motion in orbits with rotating apsidal lines can lead to rapid accumulation of particles and the formation of small bodies. At small eccentricities of orbits, rapid clearing of the annular zone of the protoplanetary disk will occur. At critically large eccentricities, the orbits will become orbits of accretion of bodies onto the protostar core. Let us denote such a ring-shaped zone by  $K_w$  and determine the dynamic viscosity in such a ring. As a result of inelastic collisions, the Keplerian orbits of dust particles are transformed into elliptical orbits with rotating apsidal lines. The speed of orbital

motion in  $K_w$  disks is determined using the following formulas (Abdulmyanov 2022):

$$v_r = \frac{dr}{dt} = \frac{ekh \sin(k\phi)}{p}, v_\phi = r \frac{d\phi}{dt} = \frac{kh}{r}, r = \frac{p}{1 + e \cos(k\phi)}, \quad (1)$$

where  $k$  is a constant characterizing the rotation of the line of apses of the elliptical orbit. Substituting the expressions  $v_r, v_\phi$  into the viscosity formula, we obtain the dynamic viscosity in  $K_w$  disks:

$$\eta(r, t) = \left[ \left( \frac{k^2 h^2}{r^2} + \frac{gM}{r} - \frac{ek^2 h^2 \cos(\phi)}{pr} - \frac{k^2}{3} \left( \frac{he \sin(\phi)}{p} \right)^2 \right) \rho - r^{1/4} \frac{\partial \rho}{\partial r} \right] \frac{3p}{4ekh \sin(\phi)}. \quad (2)$$

At  $k = 1$  there will be no rotation of the apsidal line. In this case, the viscosities in the  $K$  and  $K_w$  disks will be the same. The second mechanism is the mechanism of dust accumulation near the equilibrium orbits of the circular limited three-body problem. In this case, the movement of dust particles will occur along librational orbits of the limited three-body problem (Garfinkel 1976, 1977; Abdulmyanov 2001). The value of the resonance parameter  $\alpha$ , at which a small body leaves a given resonance, determines the outer boundary of the ring. The ring, which is determined by the zone of action of the orbital resonance of the form  $(s + q)/s$ , will be denoted by  $K_r$ . Let us determine the dynamic viscosity of motion in the ring  $K_r$ . The components of the velocity vector  $V = (v_r, v_\phi)$  in this case will be determined using the following velocity formulas (Abdulmyanov 2022):

$$v_r \approx 2mG^3 f'_0(\lambda^*) + eG^4 \sin(\phi) \left[ \frac{1}{(G + \Gamma)^3} - \frac{s + q}{s} n_1 \right], v_\phi \approx G^2 \left[ \frac{1}{(G + \Gamma)^3} - \frac{s + q}{s} n_1 \right]. \quad (3)$$

Substituting the expressions  $v_r, v_\phi$  into the viscosity formula (Abdulmyanov 2022), we obtain:

$$\eta(r, t) = \frac{3}{4} \left[ \frac{gM\rho}{r} - r^{1/4} \frac{\partial \rho}{\partial r} \right] / \left[ 2mG^3 f'_0(\lambda^*) + eG^2 \sin(\phi) \left[ \frac{1}{(G + \Gamma)^3} \right] \right], \quad (4)$$

where  $g$  is the gravitational constant,  $M$  is the mass of the Sun (protostar),  $m$  is the mass of Jupiter (ring),  $\phi, e$  is the true anomaly and eccentricity of the Keplerian orbit of the asteroid (dust particles). The function  $f_0(\lambda^*)$  characterizes the libration motion of asteroids (Abdulmyanov 2001), the angle  $\lambda^* = \lambda - n_1(s + q)/s$  is determined as a function of time in the form of a series of Jacobi functions (Abdulmyanov 2022),  $\lambda, \phi, n_1$  – average longitude, true anomaly of dust particles and the average motion of Jupiter,  $s, q$  – integers that determine the commensurability of the average motions of

Jupiter and the dust particle. According to formula 2, for circular orbits the viscosity will be infinitely large. It is possible that such viscosity in equilibrium circular orbits was the reason for the formation of co-orbital asteroids of Jupiter. An increase in orbital eccentricity occurs in critical cases: in the case of a small body leaving the orbital resonance or in the mode of accretion onto the core of a protostar. To model the exit of a small body from orbital resonance, it is necessary to take into account the influence of the rate of accretion of interstellar matter on the protostar core or the rate of dust sedimentation on the equatorial plane of rotation of the protostar. For this, the resonance parameter in the viscosity formula 4 must evolve and be a function of time. The cases of accretion onto the “embryo” of a planet and accretion onto the core of a protostar should also be distinguished. Data from observations of gas and dust disks of young stars (Garufi et al. 2017; Garufi 2017; Facchini 2017; Ruge et al. 2013) show the existence of both ring-shaped and spiral fragmentation. In order to study the process of formation of small bodies in ring-shaped fragments, it is necessary to use modern computer tools for numerical integration of gas dynamics equations. In this case, all parameters in formulas 2 and 4 will be variable and will be determined by the magnitude and direction of the velocity vector at the current time. The spiral orbits of planets are the result of gravitational disturbances. The deviation of the orbits of planets from elliptical is associated with mutual perturbations of the planets, and is also the result of the action of the accretion mechanism at an early stage of planet formation. The total result of the influence of these mechanisms can be quantitatively expressed in wavelengths GW (Table 1). If we compare the

**Table 1.** The value of the constant ( $h^2$ ) of the area integral for  $K_w$  orbits and the length of gravitational waves (GW) of the Solar system.

Planets	$a$ (AU)	$e$	$\Delta\pi(\text{sec})$	$h^2 10^3$	$T$	$L_{\text{GW}}, \text{m}$
Mercury	0.3871	0.2056	+0.93	10.4741	0.2408	$3.56 \times 10^{11}$
Venus	0.7233	0.0068	+0.84	14.6302	0.6151	$6.8 \times 10^{11}$
Earth	1.0000	0.0167	+1.03	17.2005	1.0000	$9.4 \times 10^{11}$
Mars	1.5237	0.0934	+1.1	21.1432	1.8801	$1.43 \times 10^{12}$
Jupiter	5.2028	0.0485	+0.97	39.2309	11.8622	$4.89 \times 10^{12}$
Saturn	9.5388	0.0556	+1.18	53.1394	29.4577	$8.97 \times 10^{12}$
Uranus	19.1818	0.0473	+0.96	75.5387	84.0152	$1.81 \times 10^{13}$
Neptune	30.0580	0.0086	+0.48	94.6551	164.788	$2.84 \times 10^{13}$
Pluto	39.5200	0.2530	+0.84	105.6321	247.696	$3.62 \times 10^{13}$

GW wavelengths with those currently observed from  $10^6$  m to  $10^{-14}$  m, it is easy to see that the GW waves (from  $3.62 \times 10^{13}$  m to  $3.56 \times 10^{11}$  m) are on the left flank of the interval wavelengths [ $10^6$  m;  $10^{-14}$ ] m. In this regard, the planetary system

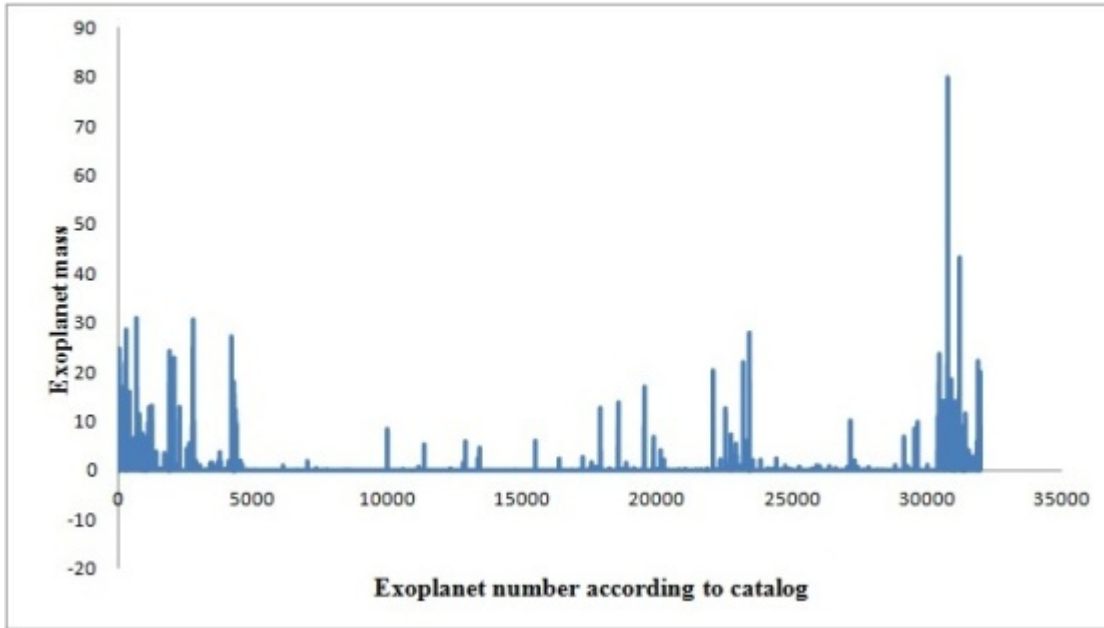
of the Solar System is the best object for studying gravitational waves: the system itself is an adequate and natural instrument for recording gravitational waves. The wavelength, expressed in kilometers, will be equal to:

$$L_{\text{GW}} = 29.785 \left( \frac{1 + m_1}{a} \right)^{1/2} \left[ (1 - e^2)^{1/2} + T \frac{\Delta\pi}{60} \frac{1}{360} \frac{1}{1 + e^2/2} \right] lT^*, \quad (5)$$

where  $T^*$  is the same period of movement along the elliptical orbit  $T$ , only expressed in seconds,  $l$  is the smallest number of revolutions along the elliptical orbit, after which particle  $m_1$  returns to the starting point, an orbit with a rotating periapsis. The number  $l$  is equal to the denominator of the fraction of the best approximation of real numbers by rational numbers.

### 3 Comparison of modeling results and data from the NASA Exoplanet Archive catalog

According to Fesenkov's hypothesis, the formation of stars and planetary systems occurs almost simultaneously. In this case, the star formation mechanism will influence the process of formation of celestial bodies. The influence of the star formation mechanism on the dynamics in gas-dust disks can be considered using the general gas dynamics equations. Namely, the dynamic viscosity  $\eta(r, t)$ , which was obtained using these equations, can be used to solve the problem of angular momentum deficiency in the Sun and determine the reason for the appearance of hot giant planets near stars of other planetary systems. According to Fig. 1 the masses of many exoplanets are an order of magnitude greater than the mass of Jupiter. However, many of them have semimajor axes similar to the orbits of the Earth group planets. For example, exoplanets (63) 51 Peg b, (188–191) CoRoT–1 b, (202) CoRoT–12 b, (207) CoRoT–13 b and many others have large masses and small eccentricities ( $e < 0.3$ ). Exoplanets (132) 70 Vir b, (198) CoRoT–11 b, (211) CoRoT–16 b, (232) CoRoT–20 b and others have large masses and large eccentricities ( $e > 0.3$ ). In parentheses before the names of exoplanets are their serial numbers in the NASA Exoplanet Archive (2022) catalog of exoplanets Exoplanet Catalog (<http://exoplanetarchive.ipac.caltech.edu>). According to the NASA Exoplanet Archive catalog most exoplanets in the catalog have orbits with small eccentricities ( $e < 0.3$ ). The close location of exoplanets with large masses to the main star according to the accretion model considered in this work is a consequence of the loss of their angular momentum as a result of the influence of dynamic viscosity. This process of transfer of angular momentum is associated with the process of formation of celestial bodies and, therefore, is irreversible. That



**Fig. 1.** Distributions of exoplanets by their masses according to the NASA Exoplanet Archive catalog, expressed in Jupiter masses.

is, the reverse migration of large celestial bodies from the central star to the periphery is unlikely or impossible. Consequently, the orbits of gas condensations and “hot Jupiters” located near stars are unstable, rapidly evolving orbits of accretion of interstellar gas onto the central star. According to the results of modeling (Abdulmyanov 2020), for solar mass stars, gravitational compression is from 50 AU up to 5 AU occurs near the boundary of the zone of gas-dynamic instability, and the modeling problem becomes an ill-conditioned problem. In this case, close distances between the planets Pluto–Neptune–Uranus–Saturn ( $d= 9.5; 10.8; 9.7$  AU) cannot also be characteristic of other planetary systems, even if the central star has a mass equal to the mass of the Sun. For such distances to form, it is necessary that accretion rates be exactly the same as for the early Solar System. However, such a coincidence can only be accidental and unlikely. That is, the typicality of the solar planetary system, in comparison with other planetary systems, is not confirmed by both modeling results and observations. The results of observations of exoplanets with masses on the order of the mass of Jupiter near central stars are currently being actively analyzed in publications. A complete review of publications, results and methods of observing exoplanets can be found in the monograph by Sakhbullin 2020.

## 4 Conclusion

During gravitational compression of a protostellar cloud, the pulsational part of its density disturbances will determine the formation of a large-scale structure of gas and dust disks (Abdulmyanov 2020). In this case, the corresponding dynamic viscosity will participate in the process of disk evolution as a mechanism that transmits and distributes angular momentum. Taking this into account, in this work: 1) the dynamic viscosity of Keplerian and almost Keplerian dust disks of protostars has been determined. It has been shown (Abdulmyanov 2022) that dynamic viscosity determines changes in the accretion rate and the characteristic time of accretion. 2) The results of the analysis of exoplanet catalog data show that exoplanets, whose masses are comparable to the masses of Jupiter, could approach the central star along accretion orbits onto the star. Such migration of exoplanets may be the result of dynamic viscosity and their loss of angular momentum. 3) Gravitational waves in the range from  $3.62 \times 10^{13}$  m to  $3.56 \times 10^{11}$  m are on the left flank of the wavelength scale [ $10^6$  m;  $10^{-14}$  m].

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