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Behavior of sectorial instabilities depending on the parameters of the generalized model of a self-gravitating disk

S. Nuritdinov[1] and K. Mirtadjieva[1,2]

[1] National University of Uzbekistan named after Mirzo Ulugbek, Tashkent, 100174 Uzbekistan
[2] Astronomical Institute of the Academy of Sciences of Uzbekistan, Tashkent, 100052 Uzbekistan

Abstract. In this work we study the problem of gravitational instability in the generalized model of a nonlinearly radially pulsating disk with an anisotropic velocity diagram relative to sectoral perturbation modes. This model is a nonstationary generalization of the equilibrium self-gravitating disk of Bisnovatyi-Kogan and Zeldovich. Nonstationary analogues of dispersion equations (NADE) are obtained on the background of this generalized model for sectoral modes of perturbations. Based on the results of numerical calculations of NADE, graphs comparing the increments of instability as a function of the initial virial ratio of the system for different values of the parameters α and β, characterizing the difference and the degree of anisotropy of the nonlinearly nonstationary generalized model of the self-gravitating disk, are constructed. In particular, it is found that the development of the bar-like perturbation mode will be the same for all anisotropic models, since the NADE of this mode does not depend on these parameters α and β.

Keywords: galaxies: evolution, formation, kinematics and dynamics, structure

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1 Introduction

When studying the early stages of the evolution of disk-shaped self-gravitating systems at the initial collisionless stage of their formation, such as during the collapse of protodisks and the formation of their large-scale structures, it is crucial to identify possible types of instabilities in the considered gravitating systems. Therefore, the construction of analytically solvable nonstationary self-gravitating models and the analysis of the phenomenon of gravitational instabilities against the background of these nonlinear nonequilibrium states is currently the most important and defining link in the study of the early evolution and physics of disk-shaped gravitating systems. This justifies the relevance of this scientific research on a global level. Based on this, in the paper Mirtadjieva 2012 we have constructed a generalized phase model of a radially pulsating disk with an anisotropic velocity diagram, which is based on a nonstationary generalization of the well-known equilibrium isotropic disk model (Bisnovatyi-Kogan & Zeldovich 1970). In the following, on the basis of this generalized model of a radially nonstationary self-gravitating disk, we study the problems of gravitational instabilities of sectoral perturbation modes.

2 NADE of sectoral perturbation modes of the generalized model

In the paper Mirtadjieva 2012 we constructed a nonlinear nonstationary generalized model with an anisotropic velocity diagram of a self-gravitating disk

$$\Psi_a = C_{\alpha\beta}\frac{\sigma_0}{2\pi}\chi(D)\sum_{n=0}^{\beta}\sum_{j=0}^{2\alpha+2n}\frac{(-1)^n\beta!}{n!(\beta-n)!}\binom{2\alpha+2n}{j}(rv_\perp)^j\,(\sqrt{D})^{2\alpha+2n-j}T_{nj}, \qquad (1)$$

where α and β are integers, $T_{nj} = (2\alpha+2n-j-1)!! \cdot g/(2\alpha+2n-j)!!$, with $g = \pi/2$ for even and $g = 0$ for odd j. As is known, in order to find the criteria for instabilities of nonlinear nonstationary models with respect to a specific type of perturbation, it is necessary to derive the corresponding NADE. Note that in this work we consider the case when the perturbations are sectorial ($N = m$), where N is the main perturbation index, and m is the azimuthal wave number. Sectoral modes start with $\mathbf{N} = \mathbf{m} = \mathbf{2}$, the so-called "bar modes", which are thought to be important in the formation of bars in SB-type disk galaxies, various ellipticities, and elongated formations. The NADE for this mode of perturbation for the isotropic model was obtained in Nuritdinov et al. 2008:

$$\Lambda\ell_\tau(\psi) = \frac{3\mathrm{c}^{1-\tau}\mathrm{s}^{\tau}}{2(1+\lambda\cos\psi)^2}\left[\mathrm{G}(\psi)\ell_0(\psi) + \mathrm{Q}(\psi)\ell_1(\psi)\right], \quad \tau = 0; 1 \qquad (2)$$

where $\mathrm{G}(\psi) = \mathrm{c} - \mathrm{i}\Omega\mathrm{es}$ and $\mathrm{Q}(\psi) = \mathrm{e}^2 s + \mathrm{i}\Omega\mathrm{ec}$, with $\mathrm{s} = \sin\psi$, $\quad \mathrm{c} = \cos\psi + \lambda$, $\quad \mathrm{k} = \sqrt{1-\lambda^2}$. Multiply eq. (2) by the weight function

$$\rho(\Omega) = \mathrm{C}_{\alpha\beta}\Omega^{2\alpha}\left(1 - \Omega^2\right)^{\frac{2\beta+1}{2}} \tag{3}$$

and integrate the result over Ω in the interval from -1 to +1. As a result we get the NADE for the bar mode of the generalized model:

$$\Lambda\ell_\tau(\psi) = \frac{3\mathrm{c}^{1-\tau}\mathrm{s}^\tau}{2(1+\lambda\cos\psi)^2}\left[\mathrm{c}\ell_0(\psi) + \mathrm{e}^2\ \mathrm{s}\ell_1(\psi)\right] \tag{4}$$

The sectoral mode $\mathbf{N} = \mathbf{m} = \mathbf{3}$. The manifestations of this "triangular mode" are especially noted in the book Binney & Tremaine 1987. Applying the above averaging method to eq. (31) with $\nu = 0$ from our work we obtain the following NADE for this mode against the background of the generalized model:

$$\Lambda\eta_\tau(\psi) = \frac{15c^{2-\tau}s^\tau}{8(1+\lambda\cos\psi)^4}B(\psi).(\tau = \overline{0,2}). \tag{5}$$

Here $B(\psi) = [(\mathrm{c}^2 + \mathrm{e}^2\ \mathrm{s}^2\mathrm{p})\,\eta_0(\psi) + 4\mathrm{e}^2\,\mathrm{cs}\eta_1(\psi) + (\mathrm{e}^2\ \mathrm{s}^2 - \mathrm{c}^2\mathrm{p})\,\eta_2(\psi)]$, $\mathrm{p} = \frac{2\alpha+1}{2(\alpha+\beta+2)}$. Now consider the case $\underline{\mathbf{N} = \mathbf{m} = 4}$, which corresponds to "boxy" perturbations. Observations show that there are galaxies with isophotes of density in the form of a boxy shape (a typical example is NGC 1600). Therefore this mode is also of certain interest. Averaging formulas (33) at $\nu = 0$ in the work Mirtadjieva 2009 with Ω the weight function (eq. 3), we obtain the following dispersion relation for this perturbation mode for the generalized model:

$$\Lambda\mu_\tau(\psi) = \frac{35c^{3-\tau}\tau}{16(1+\lambda\cos\psi)^6}F(\psi), (\tau = \overline{0,3}), \tag{6}$$

where

$$F(\psi) = \left(c^3 - 3e^2css^2p\right)\mu_0(\psi) + \left(e^2c^2s + 2e^2c^2sp - e^4s^3p\right)\mu_1(\psi) +$$
$$+ \left(e^4css^2 + 2e^4c^2p - e^2c^3p\right)\mu_2(\psi) + \left(e^6s^3 - 3e^4c^2sp\right)\mu_3(\psi).$$

3 Conclusions

As can be seen, eq. (4) for the bar mode does not depend on the α and β parameters. This means that the NADE for this mode remains unchanged regardless of whether the model is isotropic or anisotropic in terms of velocity components. It should be noted that the NADE (eq. 5) for the triangular mode does not lend itself to analytical consideration. It has therefore been studied numerically using the method of stability of periodic solutions (Malkin 2016). For a comparative analysis of the stability of the anisotropic models (eq. 1) with respect to this perturbation mode, using numerical calculations of NADE (eq. 5), we have constructed graphs of the dependence of the instability increments on the initial virial ratio of the models for different values of the parameters α and β (Fig. 1). From Fig. 1 it can be seen that an increase in the

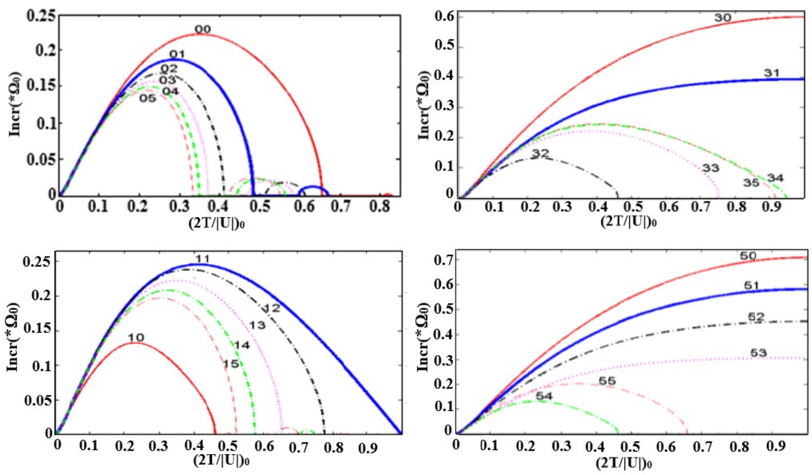

Fig. 1. Comparison of instability increments as a function of the initial virial ratio of models (1) with different values of α and β for the (3;3) mode of sectoral perturbations. The first number in the graphs is α, the second is β.

parameter α has a destabilizing effect during the evolution of this mode against the background of the studied anisotropic models, while the parameter β, on the contrary, plays a "stabilizing" role. However, the observed pattern for β does not fully hold for small values of the parameter α. Moreover, as the value of α increases, the range of the initial virial ratio, where the triangular structure forms against the background of anisotropic models, occupies the entire range of its possible values. Numerical calculations of the NADE (eq. 6) for the boxy mode show that the parameters α and β, which characerize the nonlinear nonstationary anisotropic models of the self-gravitating disk, produce the same effects in the development of this mode as in the triangular mode of sectorial perturbations.

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