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Fire-hose instability of the torsional Alfven oscillation as a mechanism of plasma layering in an experimental coronal solar loop-type magnetic arch

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Abstract. The compact experimental setup “Solar Wind” (IAP RAS) is designed for modeling plasma processes in a magnetic arch of the coronal solar loop type. Arc discharges at the bases of the arch create plasma with a significantly increased ionic temperature along the magnetic force line. Optical images show that the plasma rope is stratified into two belts along the upper and lower vaults of the arch or as a near-wall cylindrical layer. The system stratifies when the longitudinal ionic pressure at the top of the loop is 2 times the magnetic pressure. The indicated pressure ratio coincides with the threshold value for the development of the fire-hose instability of the Alfven wave in a plasma with temperature anisotropy. We propose a variant of the growing torsional Alfven oscillation as a mechanism for the formation of a cylindrical layer along the plasma tube wall.

Keywords: instabilities; magnetohydrodynamics; plasmas; Sun: coronal mass ejections, filaments, prominences

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1 Introduction

The solar corona is structured into a system of plasma arches (loops) whose characteristic length varies from the height of a homogeneous chromosphere to a value on the order of the solar radius (Zaitsev & Stepanov 2008). Magnetic loops are waveguides and resonators for the Alfven wave, magnetic and ionic sound (Stepanov et al. 2012). The small diameter of the loops compared to their length limits the plasma pressure inside them at a level well below the magnetic pressure. An increase in plasma pressure to a higher value is accompanied by flute instability (of Alfven-type surface wave) with plasma ejection at the apex of the arch.

In turn, the solar wind is characterized by a higher plasma pressure compared to the magnetic pressure, as well as by anisotropic ionic temperature relative to the direction of the local magnetic field (due to different degrees of plasma expansion along and across the magnetic force line). As a possible mechanism of isotropization of ion temperature, Yoon et al. (2019) consider the fire-hose and mirror instabilities (turbulence) of the Alfven and magnetosonic waves. The fire-hose and mirror instabilities are essentially a Weibel instability modified by the magnetic field and originates from a part of the ion (or electron) current caused by the scattering of particles on an inhomogeneous wave magnetic field (Mikhailovskii 1974, Ch. 15, 18).

2 Experimental setup "Solar Wind"

The compact laboratory setup "Solar Wind" (Viktorov et al. 2019) creates a plasma system of the coronal solar loop type in a vacuum chamber with a diameter of $D = 15$ cm. The pulse solenoid coils enclose two mutually perpendicular radial ports of the vacuum chamber and form a curved magnetic tube resembling a quarter arc of a circle. A localized vacuum arc discharge on an aluminum cathode with a diameter of 10 mm is located on the central axis of each of the solenoids and injects a plasma cloud into the base of the magnetic arc. The plasma cloud expands along the magnetic force line at a velocity $v_{\mathrm{i}\|} = 15$ km/s, which is 3.5 times the ion-sonic velocity for a characteristic electron temperature of 3 eV and charge number of ions $Z = 1.7$. A 20 μs discharge generates a plasma blob with a length of 2 diameters of the vacuum chamber, or approximately 2.5 of the arch length $L_{\mathrm{loop}} = \pi D/4$. Encountering supersonic blobs (from each arch base) jointly form a plasma system with zero hydrodynamic velocity but increased ionic temperature along the magnetic force line.

Optical images with a long exposure time of 1 s (compared to the time of the system's existence) revealed the plasma rope layering into two belts along the upper and lower vaults of the arch or compression into a narrow cylindrical layer along the arch wall (Viktorov et al. 2019). Photographs were taken from the facade of the arch, so it is not possible to choose one of the stratification variants. Layering was recorded at an arc discharge current $I_{\rm gen} = 8$ kA, which corresponds to an ionic density at the base of the loop (in one stream) $n_{\rm i\,foot} = 2 \cdot 10^{14}\,(I_{\rm gen}/1\ {\rm kA})\ {\rm cm}^{-3} = 1.6 \cdot 10^{15}\ {\rm cm}^{-3}$ (Viktorov et al. 2015). The tube expands $\sqrt{\alpha} = 3$ fold from a diameter of 10 mm at the foot to about 30 mm at the apex of the arch. Increasing the cross-sectional area of the rope by $\alpha = 9$ fold reduces the ion density on the way to the top to $n_{\rm i\,top} = n_{\rm i\,foot}/\alpha = 1.8 \cdot 10^{14}\ {\rm cm}^{-3}$. The longitudinal thermal pressure from the two plasma flows reaches the level $P_{\rm pl\|} = 2 n_{\rm i\,top} m_{\rm i} v_{\rm i\|}^2 = 3{,}7 \cdot 10^4\ {\rm dyn/cm}^2$, which is about 1.9 times the magnetic pressure $P_{\rm mag} = B_0^2/(8\pi) = 1.9 \cdot 10^4\ {\rm dyn/cm}^2$ for the unperturbed induction $B_0 = 700$ G at the same point (ion mass $m_{\rm i} = 27$ u).

The ratio of longitudinal plasma and magnetic pressures, $\beta_\| = P_{\rm pl\|}/P_{\rm mag}$, close to two, indicates the possible development of fire-hose instability of the Alfvén wave and magnetic sound (Mikhailovskii 1974, § 18.7–18.9); (Krall & Trivelpiece 1973, Ch. 3, § 10). The observed belts or cylindrical layer are strongly extended along the arch for a distance significantly larger than the plasma rope radius. Therefore, the local wave vectors of the waves (whose superposition forms a perturbation of the plasma density) are oriented almost perpendicular to the external magnetic field. For the indicated direction of the wave vector, the fire-hose instability of the magnetic sound is suppressed, whereas for the Alfven wave the threshold relation $\beta_\| = 2$ is the same for any wave vector.

3 Unstable torsional Alfven oscillation

Based on the energy principle of Bernstein et al. (1958), the perturbation of the magnetic field in vacuum (outside the plasma tube) has a stabilizing effect on magnetohydrodynamic instabilities, in particular, the fire-hose one. Therefore, we considered the azimuthally uniform (torsional) Alfven oscillation, which differs from the other wave modes by the zero radial component of the magnetic induction and therefore is completely localized inside the plasma rope without leaking into the vacuum (provided that the longitudinal plasma current integral over the cross section of the tube is zero).

The radial profile of the only nonzero – azimuthal – component of magnetic induction $\delta B_\phi(\rho)$ is quite arbitrary (as long as the wavelength in the radial coordinate ρ exceeds the ionic Larmor radius). This property is reflected in the dispersion equation for the square of the complex frequency $\omega = {\rm i}\gamma$ of the Alfven wave in a homogeneous plasma

$$\omega^2 = (c_{\rm A}^2 - v_{\rm i\|}^2)\, k_\|^2, \tag{1}$$

which contains only the component $k_\parallel$ of the wave vector parallel to the external magnetic field $\mathbf{B}_0$. Here, $c_\mathrm{A}^2 = B_0^2/(4\pi n_{\mathrm{i}\Sigma} m_\mathrm{i}) \equiv 2v_{\mathrm{i}\parallel}^2/\beta_\parallel$ is the square of Alfven velocity, $n_{\mathrm{i}\Sigma}$ is the total ion density over the streams, $\gamma = -\mathrm{i}\omega$ is the complex wave increment.

We have considered the paraxial Euler equation of ideal magnetic hydrodynamics for the azimuthal displacement $\xi_\phi(\eta, z)$ of a plasma element in a torsional Alfven oscillation as a model equation of the fire-hose instability in a longitudinally inhomogeneous magnetic tube (neglecting the tube curvature linking different azimuthal modes)

$$m_\mathrm{i} n_{\mathrm{i}\Sigma} \frac{\partial^2 \xi_\phi}{\partial t^2} = \frac{\partial}{\partial z}\left[m_\mathrm{i} n_{\mathrm{i}\Sigma}\,(c_\mathrm{A}^2 - v_{\mathrm{i}\parallel}^2)\,\frac{1}{\sqrt{B_0}}\,\frac{\partial\,(\sqrt{B_0}\,\xi_\phi)}{\partial z}\right]. \tag{2}$$

Here, the Alfven velocity c_A and the ion density $n_{\mathrm{i}\Sigma}$ depend on the coordinate z, measured from the apex of the arch along the central magnetic force line in the tube; the radial coordinate $\eta = \rho/r_\perp(z)$ is normalized by the tube radius $r_\perp(z) = r_\perp(0)\sqrt{B_0(0)/B_0(z)}$ and is therefore constant along the magnetic force line. The radial electric intensity δE_ρ and the magnetic induction perturbation δB_ϕ are related to the displacement ξ_ϕ by the equalities

$$\delta E_\rho = -\frac{B_0}{c}\,\frac{\partial \xi_\phi}{\partial t}\,, \qquad \delta B_\phi = \sqrt{B_0}\,\frac{\partial\,(\sqrt{B_0}\,\xi_\phi)}{\partial z}\,, \tag{3}$$

which reflect the dominant joint electric drift of ions and electrons at a velocity of $\partial\xi_\phi/\partial t = -c\,\delta E_\rho/B_0$ (for processes with frequency/increment below the ion cyclotron frequency) and the electromagnetic induction equation $\mathrm{rot}\,\delta\mathbf{E} = -c^{-1}\,\partial\,\delta\mathbf{B}/\partial t$ (taking into account the change of the distance ρ between magnetic lines of force inversely proportional to $\sqrt{B_0}$ when moving along the tube).

In the region of fire-hose instability near the apex of the arch ($c_\mathrm{A}(z) < v_{\mathrm{i}\parallel}$), only the difference $c_\mathrm{A}^2 - v_{\mathrm{i}\parallel}^2$ in eq. (2) varies significantly, changing its sign at points equidistant from the apex. Therefore, we approximate this value by a quadratic function

$$c_\mathrm{A}^2 - v_{\mathrm{i}\parallel}^2 \approx -v_{\mathrm{i}\parallel}^2\left(1 - \frac{2}{\beta_{\parallel\,\mathrm{top}}}\right) + c_{\mathrm{A\,foot}}^2\left(\frac{2z}{L_\mathrm{loop}}\right)^2,$$

where $\beta_{\parallel\,\mathrm{top}} > 2$ is the ratio of longitudinal plasma and magnetic pressures at the apex of the arch, $c_{\mathrm{A\,foot}} \gg v_{\mathrm{i}\parallel}$ iis the the Alfven velocity at the bases of the tube, which are $L_\mathrm{loop}/2$ away from the apex. The Euler eq. (2) takes the form of the Legendre equation for time-dependent perturbations following the exponential law with an increment γ, or complex frequency $\omega = \mathrm{i}\gamma$:

$$\frac{\partial}{\partial\zeta}\,(1 - \zeta^2)\,\frac{\partial \xi_\phi}{\partial \zeta} + \kappa^2 \xi_\phi = 0, \tag{4}$$

where the complex parameter

$$\kappa = \frac{\gamma L_{\mathrm{loop}}}{2c_{\mathrm{A\,foot}}}\,,$$

and the dimensionless coordinate

$$\zeta = \frac{2z}{L_{\mathrm{loop}}}\,\frac{c_{\mathrm{A\,foot}}}{v_{\mathrm{i}\|}\,\sqrt{1-(2/\beta_{\|\,\mathrm{top}})}}\,.$$

Equation (4) must be accompanied by a boundary condition in the form of zero displacement ξ_ϕ at the tube bases, at the points $\zeta = \pm c_{\mathrm{A\,foot}}/[v_{\mathrm{i}\|}\,\sqrt{1-(2/\beta_{\|\,\mathrm{top}})} \gg 1$ — zero tangential electric intensity eq. (3) on the metal cathode of the arc discharge. At the same time, the zeroing of the coefficient at the highest derivative in eq. (4) at the points $\zeta = \pm 1$ (at the boundary of the fire-hose instability region) requires an auxiliary regularization of the solution in the form of the equality of the displacement ξ_ϕ and its spatial derivative $\xi'_\phi \equiv \partial\xi_\phi/\partial\zeta$ (providing continuity of magnetic induction and electric intensity eq. (3)) at small, but non-zero distance ε from the singular points $\zeta = \pm 1$: $\xi_\phi(1-\varepsilon) = \xi_\phi(1+\varepsilon)$, $\xi'_\phi(1-\varepsilon) = \xi'_\phi(1+\varepsilon)$. Transition regions $|\zeta \mp 1| \le \varepsilon$ have an extent on the order of the distance traveled by an ion for a quarter of its cyclotron period $\omega^{-1}_{B\mathrm{i\,top}}$: $\varepsilon \sim c_{\mathrm{A\,foot}}/[\omega_{B\mathrm{i\,top}} L_{\mathrm{loop}}\,\sqrt{1-(2/\beta_{\|\,\mathrm{top}})}\,]$.

In this approach, exponentially increasing solutions of the hydrodynamic eq. (2) are confined in the spatial region of the fire-hose instability and tunnel outside it to a distance of the order of $\Delta\zeta \sim \varepsilon\,|\ln\varepsilon| \ll 1$. Inside the $|\zeta| < 1-\varepsilon$ instability region, the longitudinal azimuthal displacement profile $\xi_\phi(\zeta)$ (on the same magnetic force line $\eta = \mathrm{const}$) is described by the sum $\xi_\phi(\eta,\zeta) = \sum_{n=0}^{\infty} a_n(\eta)\,[\mathrm{P}_\nu(\zeta) + (-1)^n\,\mathrm{P}_\nu(-\zeta)]/2$ of Ferrers functions of the first kind P_ν with degree $\nu = n + \delta\nu$ close to integers $n = 0$, 1, 2…, and detuning $\delta\nu \sim 1/|\ln\varepsilon| \ll 1$, and on the outside – an analogous sum $\xi_\phi(\eta,\zeta) = \sum_{n=0}^{\infty} a_n(\eta)\,(-1)^{n\,(1-\mathrm{sign}\,\zeta)/2}\,Q_\nu(|\zeta|)/[2\,Q_\nu(1+\varepsilon)]$ of associated Legendre functions of the second kind Q_ν (Olver et al. 2010, Ch. 14).

Torsional modes of fire-hose "turbulence" increase with incremental time

$$\gamma_n = \frac{2c_{\mathrm{A\,foot}}}{L_{\mathrm{loop}}}\,\sqrt{\nu\,(\nu+1)} \approx \frac{2c_{\mathrm{A\,foot}}}{L_{\mathrm{loop}}}\,\sqrt{n\,(n+1)}\,, \qquad (5)$$

which is independent of the extent ΔL of the unstable region at the apex of the arch. Such an effect qualitatively follows from the dispersion eq. (1) for the increment $\gamma = -\mathrm{i}\omega$, in which the wave number is "quantized" as $k_{\|n} \sim \pi n/\Delta L \propto 1/\Delta L$. In turn, the extent $\Delta L = L_{\mathrm{loop}} v_{\mathrm{i}\|}\,\sqrt{1-(2/\beta_{\|\,\mathrm{top}})}/c_{\mathrm{A\ foot}}$ is linearly proportional to the coupling coefficient $\sqrt{v^2_{\mathrm{i}\|} - c^2_{\mathrm{A\,top}}}$ between the increment and the wave number in the relation eq. (1).

According to the expression eq. (5), the torsional modes of fire-hose "turbulence" most extended along the magnetic arch are formed in the typical time $\gamma^{-1} \sim L_{\rm loop}/(2c_{\rm A\,foot})$, during which the Alfven wave travels from the base to the top of the arch. At the same order of plasma and magnetic pressures, the indicated time is of the order or shorter than the interaction time of plasma blobs in the experimental setup (see Section 2), which allows us to observe this layer structure.

4 Summary

The plasma system of the coronal solar loop type in the compact experimental setup "Solar Wind" (IAP RAS) is characterized by an elevated ionic temperature along the magnetic force line. This feature gives rise to fire-hose instability of the Alfven wave – a collisionless mechanism of temperature isotropization. The unstable torsional Alfven oscillation is capable of redistributing the plasma from the axis to the wall of the plasma tube (at a high ramp-up rate – of the order of the ion cyclotron frequency) and thereby creating the stratification observed in the experiment in the form of a near-wall cylindrical layer.

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