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Electric currents in the solar corona and the kink instability of magnetic flux rope

Yu. Tsap[1], A. Stepanov[2] and Yu. Kopylova[2]

[1] Crimean Astrophysical Observatory of the Russian Academy of Sciences, Nauchny, 298409 Russia
[2] Central Astronomical Observatory at Pulkovo of the Russian Academy of Sciences, Pulkovskoje ave. 65/1, St. Petersburg, 196140 Russia

Abstract. We found the dispersion equation for magnetohydrodynamic kink oscillations of a force-free magnetic flux rope with uncompensated longitudinal electric current under the conditions of the solar corona using the energy method and the thin magnetic flux tube approximation. It is shown that the eigenfunctions, along with the eigenvectors, impose additional restrictions on the stability conditions of the kink instability of a flux rope. This allows us to obtain not only the necessary, but also a sufficient condition for stability. The observed weak twist of coronal loops with a small (< 2–3) number of the turns of magnetic field lines around the axis indicates the dominance of unshielded magnetic flux rope in the corona of the Sun, in which the longitudinal electric currents do not exceed 10^{11}–10^{12} A.

Keywords: magnetohydrodynamics (MHD), Sun: corona, flares, coronal mass ejections (CMEs), oscillations

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1 Introduction

It is believed that electric currents responsible for the non-potential component of the coronal magnetic field are concentrated in coronal loops, which can be modeled at least in some cases as the magnetic flux ropes. In addition to the flares, they can cause coronal mass ejections, which are the most geoeffective solar events. Despite this, there is still no clear understanding of the structure and characteristic values of coronal electric currents. In particular, it has not been possible to determine whether the currents in the corona are neutralized (shielded) or non-neutralized (unshielded), as is the case in the solar photosphere (Tsap et al. 2022).

There are two types of solar flares: confined and eruptive. The type of the flare depends on the possibility of developing either torus or kink instability (Jing et al. 2018). Torus instability arises primarily due to the restructuring of the magnetic field, while the kink instability depends on the degree of twisting of the magnetic field lines. Which mechanism is more important is not yet entirely clear. Note that Jing et al. (2018) did not reveal a dependence of the flare type on the degree of twisting of magnetic flux ropes for 38 flare events, 26 of which were accompanied by coronal mass ejections. In this regard, the question arises about the possible reason for such statistics. Previously, we considered the criteria for the magnetohydrodynamic (MHD) kink instability of both shielded and unshielded (laboratory pinch) magnetic flux rope based on the energy method (Tsap et al. 2020, 2022). Using the thin magnetic flux tube approximation with a sharp boundary, it was found that shielded flux tubes are stable with respect to kink modes (Tsap et al. 2020). However, the question of the stability of unshielded flux tubes could not be fully clarified, since the authors did not pay enough attention to studying the behavior of the eigenfunctions (displacement vectors) of the system of linearized equations of ideal MHD. As a result, the stability condition obtained by Tsap et al. (2022) is only necessary. Besides, we did not consider in detail the behavior of modes with different signs of the azimuthal wave numbers $m = \pm 1$.

The aim of this paper is to derive a dispersion relation for the kink modes of unshielded magnetic ropes (coronal loops) using the energy method and to investigate the limitations imposed by the boundary conditions at the bases of the loops in the light of recent statistical studies (Jing et al. 2018).

2 Electrical currents and coronal loop twisting

Let us consider the relationship between the electric current and the twisting of coronal loops, modeled as straight, cylindrically symmetric, unshielded magnetic flux ropes with a cross-sectional radius of a, assuming a sharp plasma–plasma boundary.

According to Ampere's law, using cylindrical coordinate system (r, φ, z) and standard notation, the longitudinal current density can be represented as

$$j_z = \frac{c}{4\pi}\frac{1}{r}\frac{\partial}{\partial r}\left(rB_\varphi\right). \tag{1}$$

Multiplying Equations (1) by area elements $2\pi r dr$, after integration we have

$$I_z = I_{zi}(r \to a) + I_{ze}(r \to a) = \frac{cB_\varphi(r)r}{2}. \tag{2}$$

Equation (2) assumes that the total electric current I_z of the shielded magnetic rope, when the azimuthal field components inside (i) and outside (e) are equal to zero, i.e. $B_\varphi(a) = B_{\varphi i}(r \to a) = B_{\varphi e}(r \to a) = 0$.

Unlike the shielded flux rope, the unshielded flux rope with the uncompensated electric current ($I_z \neq 0$), for which $B_\varphi(a) = B_{\varphi e}(r \to a) = B_{\varphi i}(r \to a)$ and $I_{ze}(r \to a) = 0$, can effectively compress the plasma because the equation of the total pressure balance takes the form (Solovev & Uralov 1979)

$$\langle p_i \rangle + \frac{\langle B_{zi}^2 \rangle}{8\pi} = p_e(a) + \frac{B_{ze}^2(a)}{8\pi} + \frac{B_\varphi^2(a)}{8\pi}. \tag{3}$$

Here p is the gas pressure, B_z and B_φ are the longitudinal and azimuthal magnetic fields, respectively (in the local cylindrical system of coordinates). The brackets denote the values averaged over the cross-section of radius a.

Let us assume that the values of coronal and photospheric electric currents are within the same order of magnitude, i.e. the longitudinal electric currents of twisted coronal loop are $I_{zi} = (1-3)\times 10^{11}$ A, which correspond to $3\times 10^{20} - 10^{21}$ statA. Therefore, according to (2) for $a = (1-3)\times 10^8$ cm we obtain

$$B_\varphi(a) = 2I_z/(ca) = 60 - 700 \text{ G}. \tag{4}$$

Using eq. (2) for the total twist angle inside a flux rope at $r = a$ we find

$$\Phi(a) = \frac{LB_{\varphi i}(a)}{aB_{zi}(a)} = \frac{2LI_z}{ca^2 B_{zi}(a)}. \tag{5}$$

For $a = (1-3)\times 10^8$ cm, $L = 3\times 10^9$ cm, $B_{zi}(a) = 300$ G, and $I_z = (1-3)\times 10^{11}$ A, from Equation (5) we get $\Phi(a) = 0.6$–66. These values of the twist angle correspond to the number of turns of the magnetic field lines around the axis of the flux rope $R \approx \Phi(a)/2\pi = 0.1$–$10$.

3 Eigenvector functions and twist instability

Let's consider an axisymmetric flux rope (cylinder) with the unperturbed tube axis, which is parallel to the z-axis with the magnetic field

$$\mathbf{B} = \begin{cases} (0, B_{\varphi i}(r), B_{zi}(r)), \ r \leqslant a; \\ (0, B_{\varphi e}(r), B_{ze}(r)), \ r > a. \end{cases} \tag{6}$$

Using standard notation, it can be easily shown that that eigenvalues $\Lambda = \Omega^2$ of the magnetic configuration with a sharp plasma–plasma boundary separating the inner and outer regions by the surface σ can be found from the following expressions (Tsap et al. 2020):

$$\Lambda = \frac{2W}{\int\limits_{V_i} \rho \mathbf{s}^2 dV + \int\limits_{V_e} \rho \mathbf{s}^2 dV} = 2\frac{W_i + W_e + W_\sigma}{\int\limits_{V_i} \rho \mathbf{s}^2 dV + \int\limits_{V_e} \rho \mathbf{s}^2 dV} \, . \tag{7}$$

Here

$$W_i = \frac{1}{2}\int\limits_{V_i} K\, dV, \quad W_e = \frac{1}{2}\int\limits_{V_e} K\, dV, \quad W_\sigma = \frac{1}{2}\oint\limits_{\sigma} \frac{d\langle P\rangle}{dn} s_n^2(\mathbf{r})\, d\sigma, \tag{8}$$

$$K = \frac{\delta B^2}{4\pi} + \frac{\mathbf{j}}{c}(\mathbf{s}\times\delta\mathbf{B}) + \gamma p (\nabla \mathbf{s})^2 + (\mathbf{s}\nabla p)\nabla\mathbf{s} + \mathbf{g}\mathbf{s}\nabla(\rho\mathbf{s}),$$

$$P = p + B^2/8\pi, \quad \langle P\rangle = P_e - P_i.$$

For the kink mode ($m = \pm 1$), neglecting gravity and using the procedure of minimization, we can find the radial displacement s_r inside a flux rope in the long-wavelength limit ($ka \ll 1$) when the plasma parameter $\beta \ll 1$ is equal to $s_0 = \mathrm{const}$ (Tsap et al. 2020).

If the magnetic field outside a flux rope ($r > a$) is potential ($\nabla \times \mathbf{B_e} = 0$), we can take

$$B_{\varphi e} = A/r, \quad B_{ze} = \mathrm{const}, \tag{9}$$

where $A = \mathrm{const}$. In this case, due to minimization of W_e the radial component of eigenvector is (Tsap et al. 2022)

$$s_r(r) = s_0 \frac{kB_{ze}a^2 + mA}{kB_{ze}r^2 + mA} = s_0 \frac{a}{r}\frac{kaB_{ze} + mB_{\varphi e}(a)}{krB_{ze} + mB_{\varphi e}(r)} \, . \tag{10}$$

Equations (9) and (10) assume the following important restrictions

$$|ka| B_{ze} > B_{\varphi e}(a) \ \text{or} \ |ka| > B_{\varphi e}(a)/B_{ze} \, . \tag{11}$$

In terms of eq. (7), (8), and (10), we can find the dispersion equation in the form

$$\Omega^2 = \frac{k^2}{4\pi}\frac{B_{zi}^2 + B_{ze}^2 + 2mB_{\varphi e}(a)B_{ze}/(ka)}{\rho_i + \rho_e + m\rho_e B_{\varphi e}/(kaB_{ze})}. \qquad (12)$$

Equation (12) suggests that the kink modes ($m = -1$, $k > 0$) and ($m = 1$, $k < 0$) as well as ($m = -1$, $k < 0$) and ($m = 1$, $k > 0$) have the same eigenvalue Ω.

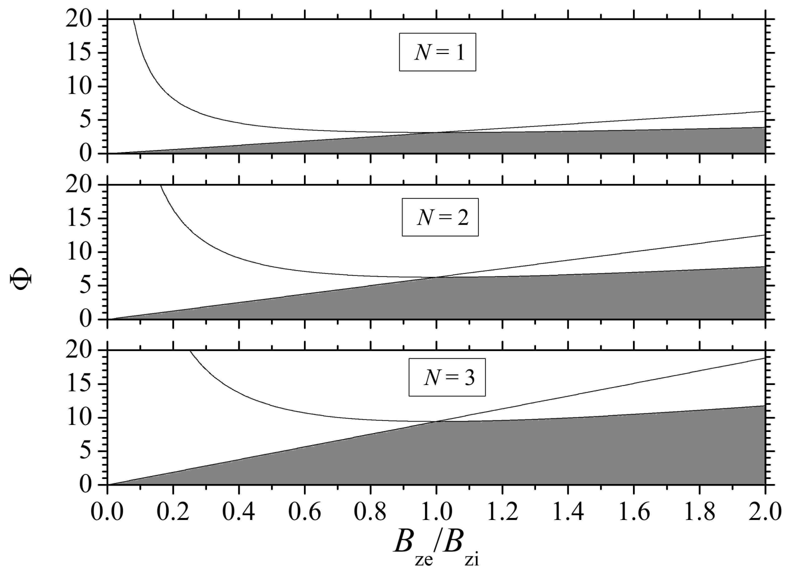

Fig. 1. The dependence of possible values of the twist angle of magnetic field lines Φ on the ratio of the longitudinal components of the magnetic field inside and outside the flux rope B_{ze}/B_{zi} for $N = 1, 2, 3$ and based on formula (16). The shaded area corresponds to the stability area.

Taking into account the condition that the footpoints of the coronal magnetic loops are frozen into photosphere and setting $|k| = \pi N/L$, where $N = 1, 2\ldots$ are the natural numbers, from eq. (5), assuming $B_{ze} = B_{ze}(a)$, we get

$$\Phi_e(a) = LB_{\varphi e}(a)/[aB_{ze}(a)] < N\pi. \qquad (13)$$

In turn, from the stability condition, according to which the numerator in the dispersion relation (12) must be positive, we have

$$\Phi(a) = \frac{LB_{\varphi i}(a)}{aB_{zi}(a)} < \frac{N\pi}{2}\frac{B_{zi}^2(a) + B_{ze}^2(a)}{B_{\varphi e}(a)B_{ze}(a)}\,\frac{B_{\varphi i}(a)}{B_{zi}(a)}\,. \qquad (14)$$

The inequalities (13) and (14) are the necessary and sufficient conditions the for the magnetic flux rope stability with respect to kink modes. In case of unshielded rope, when $B_{\varphi i}(a) \approx B_{\varphi e}(a)$, inequality (14) reduces to the form

$$\Phi(a) < \frac{N\pi}{2} \frac{B_{zi}^2(a) + B_{ze}^2(a)}{B_{ze}(a) B_{zi}(a)} \,. \tag{15}$$

Note that for $B_{zi}(a) \approx B_{ze}(a)$ and $N = 2$ from (15) we get the well-known Kruskal–Shafranov stability criterion: $\Phi(a) < 2\pi$.

For the general case, according to (13) and (15), the conditions of stability are

$$\Phi(a) < N\pi \begin{cases} B_{ze}(a)/B_{zi}(a), \\ (B_{zi}^2(a) + B_{ze}^2(a))/(2B_{zi}(a)B_{ze}(a)). \end{cases} \tag{16}$$

Figure 1 shows the stability region (shaded in gray) of a magnetic flux rope at $N = 1, 2, 3$. It is seen that the total twist angle of the magnetic field lines Φ can hardly significantly exceed 12 radians, i.e. the maximum number of turns $R \approx 12/2\pi \approx 2$.

4 Summary

We have shown that coronal loops with uncompensated electric current cannot be strongly twisted ($\Phi \lesssim 10$) due to the development of kink instability. This also suggests that the electric currents cannot significantly exceed $10^{11} - 10^{12}$ A in the solar corona. Thus, the twist angle of the magnetic field lines can serve as an observational criterion for identifying shielded and unshielded coronal flux ropes. At the same time, the lack of dependence of compact and eruptive solar flares on the degree of twisting of magnetic flux ropes can be explained by the coexistence of magnetic flux ropes of both types in the solar corona. However, we do not exclude that the excitation of eigenoscillations of coronal loops with footpoints, which are frozen-in to the photosphere, faces difficulties because of the dependence of eigenfrequency Ω on the signs of m and k (Terradas & Goossens 2012).

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