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Gravitational influence of the Moon on the Earth's poles movement

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Abstract. For a qualitative description of the oscillations of the Earth's pole, a simplified analytical model is derived, showing the presence of the frequency of periodic gravitational external action in the coordinates of the pole. As part of the refinement of the forecast of the pole movement, long-period impacts on the pole from the Moon were found and identified. Among the main characteristics of the Moon's orbit, the rotation of the node of the lunar orbit and the rotation of the perigee were chosen from the project The JPL Horizons 2024. The study used a number of C01 observational and measurement data from the International Earth Rotation Service (IERS). The observations were processed over a 122-year interval, starting in 1900 and up to 2023. In this work, as a result of numerical processing of C01 observational data, it was possible to identify quasi-stationary harmonics with the frequency of rotation of the node of the lunar orbit (as well as with a doubled frequency) and with the frequency of rotation of the perigee. The found harmonics are phase-coordinated with the positions of the node of the lunar orbit and its perigee. This confirms the existence of a mechanism of gravitational influence of the Moon (as well as other massive bodies of the Solar System) on the fluctuations of the Earth's pole. The amplitudes of these harmonics show the scale of the gravitational influence from the Moon.

Keywords: astrometry; celestial mechanics; Moon; Earth

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1 Introduction

Under the assumption that the Moon provides a gravitational influence on the movement of the Earth's poles, long-period variations of the Moon's trajectory, the rotation of the node of the lunar orbit and the rotation of the perigee, were chosen as a reason. It was previously shown that long-period lunar harmonics are present in variations of the amplitudes of the Chandler and annual components (Perepelkin et al. 2019, 2022).

2 Analytical model

We will show how the frequency of the external periodic gravitational influence arises in a change in the coordinates of the Earth's pole.

The analysis of pole oscillations is similar to the study of the gyroscope axis motion under the action of disturbing torques, taking into account viscoelastic deformations of the Earth. The Earth and the Moon make mutual translational and rotational motion around the barycenter, which, in turn, moves in an elliptical orbit around the Sun under the action of gravitational forces according to Kepler's law. Thus, in a first approximation, the oscillations of the pole obey the kinematic and dynamic Euler equations with a variable inertia tensor (Akulenko et al. 2005; Kumakshev 2018):

$$
\begin{gathered}
J\dot{\omega} + \omega \times J\omega = M, \quad \omega = (p, q, r)^T, \quad J = J^* + \delta J, \quad J^* = \text{const}, \\
J^* = diag(A^*, B^*, C^*), \quad \delta J = \delta J(t), \quad \|\delta J\| \ll \|J^*\|,
\end{gathered} \tag{1}
$$

Here ω is the angular velocity vector, in the system of the principal axes of the body, with respect to which the inertia tensor is diagonal. Since the Earth deforms when it moves in orbit under the influence of gravitational forces, it is assumed that these main axes of inertia J^* are average values, to which a small value δJ is added. The additional terms in the equation that arise in this case are considered small and neglected.

The vector of the perturbing torques M has a complex structure. This includes, among other things, external gravitational moments. Euler's kinematic equations with respect to the orbital coordinate system have the following form:

$$
\begin{gathered}
\dot{\theta} = p\cos\varphi - q\sin\varphi - \omega_0(\nu)\sin\psi, \quad \dot{\nu} = \omega_0(\nu) = \omega_*(1 + e\cos\nu)^2, \\
\dot{\psi} = \frac{p\sin\varphi + q\cos\varphi}{\sin\theta} - \omega_0(\nu) ctg\theta\cos\psi, \qquad e = 0.0167, \\
\dot{\varphi} = r - (p\sin\varphi + q\cos\varphi) ctg\theta + \omega_0(\nu)\frac{\cos\psi}{\sin\theta},
\end{gathered} \tag{2}
$$

Here $\nu(t)$ is the true anomaly, e is the eccentricity of the orbit, ω_* is a constant determined by gravitational and focal parameters.

The [1-2] system is quite difficult to apply in practice, but important qualitative conclusions can be drawn based on it. Let's show this by taking into account the gravitational influence from the Sun. To do this, we write out the term M_q^s responsible for this effect, which is included in the vector of the disturbing torque M:

$$\begin{aligned} M_q^s &= 3\omega^2[(A^* + \delta A - (C^* + \delta C))\gamma_r\gamma_p + \delta J_{pq}\gamma_r\gamma_q + \delta J_{pr}(\gamma_r^2 - \gamma_p^2) - \delta J_{rq}\gamma_p\gamma_q], \\ \omega &= \omega_*(1 + e\cos\nu)^{3/2}, \quad \gamma_p = \sin\theta\sin\varphi, \quad \gamma_q = \sin\theta\cos\varphi, \quad \gamma_r = \cos\theta, \end{aligned} \tag{3}$$

Here γ_p, γ_q and γ_r are the guiding cosines of the gravitational torque vector relative to the associated system. To calculate $M_{p,r}^s$ a cyclic permutation of the indices p, q, r is performed. As a first approximation, we integrate the equations [2]

$$\begin{gathered} r = r^0, \quad \varphi \approx rt + \varphi^0, \quad \nu \approx \omega_* t + \nu^0, \quad \cos\theta(\nu) = a(\theta^0, \psi^0)\cos\nu, \\ \theta(0) = \theta^0 = 66°33', \quad 0.4 \le a \le 1, \quad 0 \le \psi^0 \le 2\pi, \\ \cos\theta\sin\theta = b(\theta^0, \psi^0)\cos\nu + d\cos 3\nu + \ldots, \quad 0.4 \le b \le 4/3\pi, |d| \ll 1, \end{gathered} \tag{4}$$

Now the Euler equations [1], after averaging over the fast phase φ can be represented as

$$\begin{gathered} \dot p + N_p q = \kappa_q r^2 + 3b\omega_*^2\chi_p^s\cos\nu, \quad N_{p,q} \approx N \approx (0.84 - 0.85)\omega_*, \\ \dot q - N_q p = -\kappa_p r^2 - 3b\omega_*^2\chi_q^s\cos\nu, \quad p(0) = p^0, \quad q(0) = q^0, \end{gathered} \tag{5}$$

Here N is the frequency of Chandler oscillations in years, κ_p and κ_q are slowly changing average values of $\delta J_{pr}/B^*$ and $\delta J_{qr}/A^*$. The coefficients χ_p^s and χ_q^s arise as a result of averaging over φ multipliers at $\cos\nu$ in expressions for $M_{q,p}^s$. The presence of axial symmetry suggests $\chi_{p,q}^s = \chi$, where χ is found from the experimental data of the IERS[3].

The simplified analytical model [5] contains in the right part an explicit harmonic impact with the period of the Earth's revolution around the Sun (one year). Thus, in solving such differential equations, two harmonic series appears: one with the Chandler frequency N, and the other with the annual frequency of the external driving gravitational force from the Sun. This gives an idea of how the periodic influence of the driving force appears in changing the coordinates of the Earth's pole.

However, the movement of the Earth's pole has a complex trajectory and is not completely determined by two main processes: periodic with a period of one

[3] http://www.iers.org

year and quasi-periodic with an irregular Chandler frequency (Akulenko et al. 2005; Kumakshev 2018). The frequency analysis of the pole oscillations shows the presence of numerous oscillatory components with rather small amplitudes. As mentioned above, the cause of additional harmonics is an external gravitational influence.

3 Numerical study

Oscillations with frequencies that coincided with the main components of the evolution of the lunar orbit - the rotation of the node of the lunar orbit and the rotation of the perigee were selected from the trajectory of the oscillation of the Earth's pole. A number of C01 observations and measurements of pole coordinates from the International Earth Rotation Service were used. The study was conducted over an interval of 122 years, from 1900 to 2023.

3.1 The longitude of the ascending node

The longitude of the ascending node of the Moon $\varOmega$ (according to project The JPL Horizons[4]) was taken in the form:

$$\varOmega = 125.04455501^\circ - 6962890.5431''t + 7.4722''t^2 + 0.007702''t^3 - 0.00005939''t^4, \quad (6)$$

Here $t = (TT - 2451545)/36525$, and TT is the Terrestrial Time in Julian days (see the definition in IAU 2000 Resolution B1.9).

As a result of numerical processing of the C 01 series, harmonics with a frequency of rotation of the lunar orbit node $\dot{\varOmega} = 0.05373$ cycles per year, as well as a doubled frequency of $2\dot{\varOmega} = 0.107$ cycles per year were selected and shown on Fig. 1.

The harmonic $-\cos\varOmega$ is shown above; in the middle of the figure, harmonics with frequencies $\dot{\varOmega} = 0.05373$ and $2\dot{\varOmega} = 0.107$ are selected from the pole coordinates; as well as oscillations in the angle of inclination of the lunar orbit plane to the Earth's equator (below), constructed using the ephemerides of the Moon from project The JPL Horizons.

The main harmonic of the quasi-regular oscillations of the angle I is $\cos\varOmega$, which is clearly visible from the figure. When comparing the graphs, it can be seen that the harmonic with a frequency of $\dot{\varOmega} = 0.05373$, selected from the oscillations of the pole, has an extremum when the extremum is reached at the angle I. This happens at the moment when the node of the moon's orbit passes through the points of the spring or autumn equinox. The phase-by-phase alignment of these two processes proves the connection between them. It is also seen that the doubled frequency $2\dot{\varOmega} = 0.107$,

[4] https://ssd.jpl.nasa.gov/horizons/

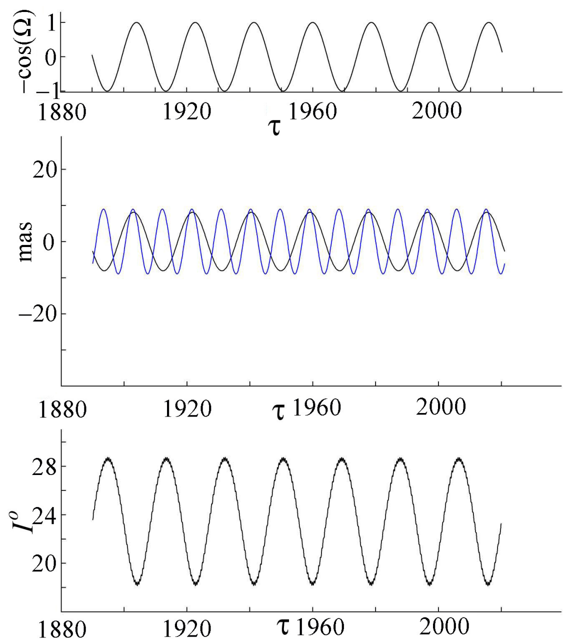

Fig. 1. The main harmonic of the longitude of the ascending node (top); harmonics selected from the coordinates of the pole (in the center); the angle of inclination of the plane of the Moon's orbit to the equator of the Earth (bottom).

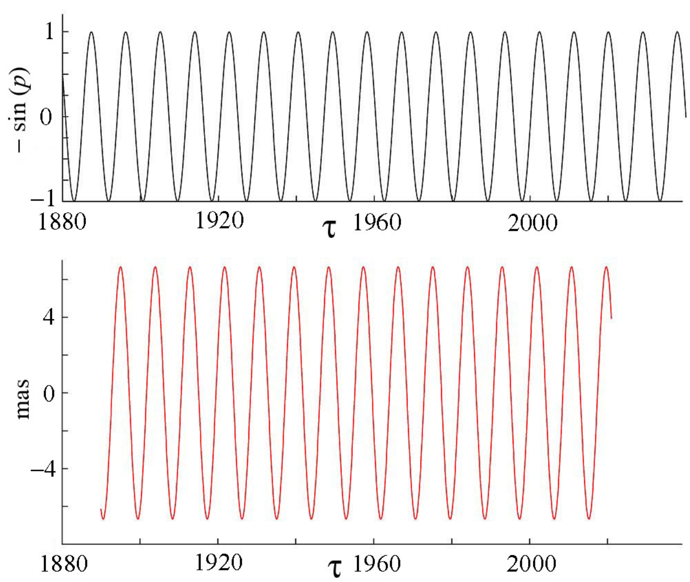

Fig. 2. The longitude of the perigee (top) and the harmonic with the same frequency, selected from the coordinates of the pole x according to the IERS data (bottom).

allocated during numerical processing of the series C 01, is phase-coordinated with the harmonic $\cos \Omega$, constructed using the ephemerides of the Moon from project The JPL Horizons.

3.2 The longitude of the perigee

According to the IERS, the longitude of the Moon's perigee p is expressed as follows:

$$p = 520.41590214^\circ + 14642259.0393''t - 38.803''t^2 - 0.044906''t^3 + 0.00020152''t^4. \quad (7)$$

In Fig. 2, this harmonic in the form of a function $-\sin p$ is depicted at the top. At the bottom of the figure there is a harmonic with the same frequency $\dot{p} = 0.11299$, but selected from the observational data of the pole oscillations for the coordinate x. It can be seen that these two oscillations are phase-coordinated. Thus, the selected harmonics from the trajectory of the Earth's pole coincide in phase with the positions of the node of the lunar orbit and its perigee. This confirms the existence of a mechanism of gravitational influence of the Moon (and other bodies of the Solar System) on the oscillations of the Earth's pole. The amplitudes of the selected harmonics from the pole coordinates show the scale of the gravitational influence of the Moon.

4 Summary

The paper examines the effect of external gravitational influence on the oscillations of the Earth's pole from the Moon. Harmonics with frequencies of long-period changes in the Moon's orbit were selected from the coordinates of the pole oscillation. A comparison of the harmonics selected from the oscillations of the Earth's pole and the evolution of the Moon's orbit showed that their phases coincide. This proves that the cause of such harmonics in the oscillations of the Earth's pole is the gravitational influence from the Moon. Taking into account such influence will increase the accuracy of forecasting the movement of the Earth's pole.

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