MODERN ASTRONOMY: FROM THE EARLY UNIVERSE TO EXOPLANETS AND BLACK HOLES

Estimation of short-period disturbances in the motion of NEAs in the presence of an inverse-square perturbing acceleration

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Abstract. Let a point of zero mass move under the influence of attraction to the central body and a small perturbing acceleration $P' = P/r^2$, where r is the distance to the attracting center, the components of the P vector are assumed to be constant in the reference system with axes directed along the radius vector, transversal and angular momentum vector. For this problem, the equations of motion in the mean elements and formulas for the conversion from osculating elements to the mean ones in the first order of smallness were previously found. The analytical solution to the averaged equations and the Euclidean (root-mean-square over the mean anomaly) displacement norm $\varrho^2 = ||dr||^2$ were also obtained earlier, where dr represents the difference between the position vectors in the osculating and mean orbits. Using these relations, it is possible to estimate the ϱ magnitude of short-period orbital disturbances arising due to the presence of a small perturbing acceleration P', as well as the d displacement of the position of the asteroid in the mean perturbed orbit from its unperturbed position after one orbital period. For 412 near-Earth asteroids (NEAs), the ϱ displacement and the d distance were calculated. An analysis of the calculation results showed that in most cases the deviation of the osculating orbit from the mean one due to periodic perturbations is small, however, cases with the ϱ displacement of the order of several tens and hundreds of kilometers have been identified. This indicates the need to take into account short-period disturbances when short-term forecasting the movement of these objects, e.g. when planning space missions.

Keywords: celestial mechanics; minor planets, asteroids: general; methods: analytical; reference systems

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1 Introduction

We considered the motion of the asteroid $\mathcal{A}$ under the influence of attraction to the central body $\mathcal{S}$ and perturbing acceleration P'. Let the P' vary inversely with the square of the distance from $\mathcal{S}$, that is, $P' = P/r^2$, where r is the distance to the attracting center. We introduced an orbital reference frame O with the origin in $\mathcal{S}$ and axes directed along the radius vector, the transversal (perpendicular to the radius vector in the osculating plane in the direction of motion), and the binormal (directed along the angular momentum vector). Let the P' be small compared to the main acceleration $\varkappa^2/r^2$, where $\varkappa^2$ is the product of the gravitational constant by mass $\mathcal{S}$, and the S, T, W components of the P vector are small and constant in the $\mathcal{O}$ frame.

Sannikova & Kholshevnikov (2019) applied an averaging procedure to the Euler-type equations of motion and obtained mean-elements motion equations and formulas for the conversion from osculating elements to the mean ones in the first order of smallness for this problem. Mean-elements motion equations at $S, T, W \neq 0$ and in various special cases were solved by Sannikova & Kholshevnikov (2020). Further, for this problem Sannikova (2024) obtained the Euclidean (root mean square over the mean anomaly) displacement norm $\varrho^2 = ||dr||^2$, where dr represents the difference between the position vectors on the osculating and mean orbits. Using these relations, it is possible to estimate the ϱ magnitude of short-period orbital disturbances arising due to the presence of a small perturbing acceleration P', as well as the d distance between the position of the asteroid in the mean perturbed orbit and its unperturbed position after one orbital period. Since the d displacement occurs due to secular drifts of elements under the influence of acceleration P', it characterizes the magnitude of the secular perturbation of the orbit. By comparing ϱ with d, one can decide on the possibility of neglecting periodic disturbances (more precisely, the differences between the osculating elements and the mean ones) and taking into account only the secular motion, which is given by the averaged equations.

2 Methods

Let us introduce the following notation: ϵ_n^0 are the osculating elements in the initial epoch t_0; $\bar{\epsilon}_n^0$ are the mean elements in the initial epoch t_0; $\bar{\epsilon}_n$ are the mean elements in the epoch $t = t_0 + dt$, where dt is one orbital period. The transformation of osculating elements into mean ones is carried out according to the formulas $\bar{\epsilon}_n^0 = \epsilon_n^0 - \delta\epsilon_n$, where $\delta\epsilon_n$ are the change-of-variables functions; their expressions are given in Sannikova & Kholshevnikov (2019) and Sannikova (2024). The norm of the difference between the osculating and mean elements is $\varrho^2 = a^2(V_1 S^2 + V_2 T^2 + V_3 W^2)/\varkappa^4$, where a is the semi-major axis, expressions for the V_i functions in the form of series in powers of the eccentricity e and in powers of $\beta = e/(1+\sqrt{1-e^2})$ are given in Sannikova (2024). We obtained the $\bar{\epsilon}_n$ elements using the solutions of the averaged equations of motion given in Sannikova & Kholshevnikov (2020) and Sannikova (2021). The $\bar{\epsilon}'_n$ elements of the mean unperturbed orbit are equal to the $\bar{\epsilon}^0_n$ except for the mean anomaly, for which $\bar{M}' = \bar{M}_0 + \bar{n}_0 dt$, where $\bar{n}_0$ is mean motion at t_0. As a result, variations of the mean elements $d\bar{\epsilon}_n = \bar{\epsilon}_n - \bar{\epsilon}'_n$ over time dt were found. We used the connection of coordinates with orbital elements, to calculate the x, y, z and x', y', z' rectangular coordinates of the asteroid on the mean perturbed and unperturbed orbits at epoch t and then, we found the distance $d = ((x-x')^2 + (y-y')^2 + (z-z')^2)^{1/2}$.

3 Results

As of April 16, 2024, the Small Body Database[1] of Jet Propulsion Laboratory (JPL) included 412 near-Earth asteroids, for which the values of non-gravitational parameters A_1, A_2 or A_3 are given, and it is indicated that when modeling non-gravitational disturbances, an inverse dependence on the square of the heliocentric distance model was used. This model of forces corresponds, for example, to the presence of the Yarkovsky effect or the pressure of sunlight. Non-gravitational radial A_1, transversal A_2 and normal A_3 parameters relate to the S, T, W components as $A_1 = S/r_0^2$, $A_2 = T/r_0^2$, and $A_3 = W/r_0^2$, where $r_0 = 1$ AU.

We estimated the ϱ and d values for these 412 objects. The following constants were used in the calculations: $\varkappa^2 = 1.32712440041279419 \times 10^{20}\,\mathrm{m^3s^{-2}}$, $1\,AU = 1.495978707 \times 10^{11}\,\mathrm{m}$, $1\,\mathrm{year} = 365.25\,\mathrm{day}$, $1\,\mathrm{day} = 86400\,\mathrm{s}$. The Table 1 shows the results of calculations. In most cases the deviation of the osculating orbit from the mean one due to periodic disturbances is small, however, cases with the ϱ displacement of the order of several tens and hundreds of kilometers have been identified, which indicates the need to take into account short-period disturbances when short-term propagating these NEAs orbits.

For 17 out of 412 NEAs ϱ exceeds 10 km, and in 4 cases ϱ is more than 100 km (see Table 1). In some cases, the magnitude of the ϱ displacement of the osculating orbit from the mean one is comparable to the d distance between the position on the mean perturbed orbit and the unperturbed position. Of the 412 NEAs, for 164 asteroids ϱ is 10% or more of d, in 6 cases more than 100%, that is, ϱ exceeds d. Thus, at times of the order of one revolution around the Sun, short-period disturbances are comparable to or exceed secular ones, so they must be taken into account when short-term forecasting the position of objects, e.g. when planning space missions.

[1] https://ssd.jpl.nasa.gov/tools/sbdb_lookup.html

Table 1. Calculation results for NEAs with the largest displacements of the osculating orbit relative to the mean one. δa and δM are the differences between osculating and mean semi-major axis and mean anomaly; $d\bar{a}$ and $d\bar{M}$ are the variations of the mean semi-major axis and mean anomaly over one orbital period; ϱ is the displacement of the osculating orbit relative to the mean orbit; d is the distance between the position of the asteroid in the mean perturbed orbit and its unperturbed position after one orbital period. Column "Note" indicates which non-gravitational parameters are given for the asteroid in the JPL database. NEAs for which ϱ exceeds d are marked with an asterisk.

Asteroid	a_0 AU	e_0	δa km	δM arcsec	$d\bar{a}$ km	$d\bar{M}$ arcsec	ϱ km	d km	Note
* 2016 NJ33	1.313	0.209	-106.885	-3.165	-4.787	0.015	651.814	40.025	$A_1A_2A_3$
* 2005 VL1	0.891	0.225	-106.944	1.490	-4.962	0.043	387.952	23.673	$A_1A_2A_3$
2006 RH120	1.033	0.025	1.987	3.950	-333.359	2.098	128.663	1639.059	$A_1A_2A_3$
* 1998 KY26	1.233	0.202	-1.624	-0.553	-1.125	0.005	104.090	6.498	$A_1A_2A_3$
2015 TC25	1.028	0.116	-13.598	0.797	-34.930	0.219	84.784	149.890	A_1A_2
2020 CD3	1.029	0.012	-0.981	-6.380	0	-1.189	70.600	876.364	A_1
* 2011 MD	1.056	0.037	-2.998	-0.262	-5.970	0.036	39.832	27.946	A_1A_2
2020 GE	1.006	0.040	2.950	-0.217	-14.442	0.093	39.247	69.241	A_1A_2
2012 LA	1.040	0.022	0.860	-1.720	12.550	-0.079	35.544	59.657	A_1A_2
2020 WY	1.020	0.029	-1.533	0.896	0	-0.545	32.094	412.848	A_1
2009 BD	1.062	0.052	-3.303	0.036	-7.756	0.047	30.256	35.581	A_1A_2
452639	2.249	0.873	-9.695	0.002	-39.013	0.113	28.799	76.567	A_2
2021 GM1	0.978	0.025	-0.066	-1.580	0	-0.505	28.505	366.538	A_1
* 2012 TC4	1.620	0.404	-12.490	0.041	-3.298	0.013	21.878	18.338	A_1A_2
* 2010 RF12	1.061	0.188	-1.974	0.128	-1.483	0.009	20.380	8.416	$A_1A_2A_3$
2016 GE1	2.065	0.520	4.806	0.002	-25.868	0.081	13.572	78.724	A_2
2013 BA74	1.754	0.445	0.374	-0.007	21.522	-0.080	10.322	73.709	A_2

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