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Systematic effect in the standard processing of gravitational-wave signals

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Abstract. We analyze a systematic effect in the standard coordinate transformation of a gravitational-wave signal from the source to the detector reference frame. It is shown that, despite the mathematically correct application of the canonical Arnowitt–Deser–Misner transform (ADM formalism), the estimated characteristic mass of the source (a binary system of merging black holes) turns out to be affected by a systematic error if the mass estimation is based on the the time derivative of the signal frequency obtained in the detector reference frame, and not in the source reference frame. This systematic error leads to a significant overestimation of distances to gravitational wave sources, which reduces the probability that large ground-based telescopes detect possible electromagnetic afterglows associated with gravitational wave sources.

Keywords: stars: black holes; gravitational waves; methods: analytical

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1 Introduction

Einstein's idea that a system of two massive bodies can emit gravitational waves (GWs) is based on the representation of the metric tensor as a linearized decomposition using a flat metric and a small perturbation propagating at the speed of light. This approximation of general relativity (called the quadrupole approximation) allows us to estimate the energy loss of a binary system due to the emission of GWs, which determines the orbital frequencies $f_{\rm orb}$, their derivatives in time $\dot{f}_{\rm orb} = df_{\rm orb}/d\tau$, and the characteristic (chirp) masses $\mathcal{M}$ of the system via the Einstein–Landau–Lifshitz quadrupole moment formalism (Landau & Lifshitz 1960; Peters & Mathews 1963).

It is important to note that the calculation of energy losses must be done in the source reference frame, using proper time τ. But in the vicinity of the GW detector, there is no information about the evolution of a binary black hole system in the proper coordinates of this system. With this in mind, we revisit the analysis of the first gravitational-wave signal GW150914, which has already been done by the Laser Interferometer Gravitational-Wave Observatory (LIGO) team in their paper (Abbott et al. 2016) dedicated to the discovery. That publication describes a standard scheme for processing GW signals, which is considered rigorous in terms of relativistic transformations of reference frames.

We also discuss the second publication by the same authors (Abbott et al. 2017), where they present an alternative analysis of the same signal GW150914 using the quadrupole approximation of general relativity (an approximation of flat spacetime). Both strict and approximate versions of the analysis give coinciding estimates for the parameters of the source of the GW signal.

In our view, this coincidence should be carefully examined, since the curvature of spacetime in the vicinity of the merger of binary black holes is very large. Therefore, the source and detector reference systems are different, which means that the parameters of the source estimated for the curved and flat spacetime metrics must also be different.

2 Quadrupole approximation

The case of GW150914. In the paper of Abbott et al. (2016), describing a relativistically strict method of processing the GW150914 signal, the following estimate of the characteristic (chirp) mass of this binary system is given: $\mathcal{M}^{\rm LVC16} = 30.4^{+2.1}_{-1.9}\, M_{\odot}$.

In their second paper, Abbott et al. (2017) used the following quadrupole approximation formula to estimate the chirp mass $\mathcal{M}$ of the system:

$$\mathcal{M}(f_{\mathrm{gw}}, \dot{f}_{\mathrm{gw}}) = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f_{\mathrm{gw}}^{-11/3}\dot{f}_{\mathrm{gw}}\right]^{3/5}, \tag{1}$$

where c is the speed of light, G is the gravitational constant, and the chirp mass is defined as $\mathcal{M} = (m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$ via the component masses m_1 and m_2. The result of this calculation, $\mathcal{M}^{\mathrm{LVC17}} \approx 37\,M_\odot$, is slightly different from the strict result, since the time intervals between the zero crossings of the signal with the abscissa used in Abbott et al. (2017) are quite noisy, and the number of points used is small. If we reduce the noise by taking the input frequencies f_{GW} and their derivatives $\dot{f}_{\mathrm{GW}}$ directly from a smoother frequency–time diagram (Yershov et al. 2023), then Eq. (1) gives us an average value of $\mathcal{M} = 30.1 \pm 2.4\,M_\odot$, almost identical to the original result.

Other cases from the Gravitational-Wave Transient Catalogue (GWTC). In addition, we can check whether other GW sources detected by the LIGO–Virgo–Kagra collaboration show that the source-to-detector coordinate transformation is incomplete, as is the case with the source GW150914. To this end, we compared the chirp masses published in the GWTC with the masses estimated using the quadrupole (post-Newtonian) approximation formula, Eq. (1). The result is shown in Fig. 1, the numerical values of the masses are presented in the appendix table in Yershov et al. (2023).

In this plot, the chirp masses $\mathcal{M}_{\mathrm{GWTC}}$ of the sources obtained by strict processing of GW signals are plotted along the vertical axis. These masses (in the coordinate system of the detector) are taken from Abbott et al. (2019); Venumadhav et al. (2020); Nitz et al. (2020); Abbott et al. (2020a,b,c). The chirp masses we have calculated using Eq. (1) for flat spacetime are plotted along the horizontal axis. We can see that the matched masses are grouped along the diagonal of the graph in the form of a linear dependence, and the Pearson correlation criterion for this dependence gives a correlation coefficient of $0.985^{+0.004}_{-0.008}$ and a p-value of $< 2.2 \cdot 10^{-16}$. This correlation is evidence that the gravitational time dilation correction was missing in the calculations of the chirp masses published in the GWTC, since Eq. (1) for flat spacetime that we have used does not include such a correction by definition.

3 Coordinate transformation

The metric coefficient g_{tt} near the horizons of black holes decreases greatly. Therefore, a remote observer should perceive the orbital motion of such a system as slowed

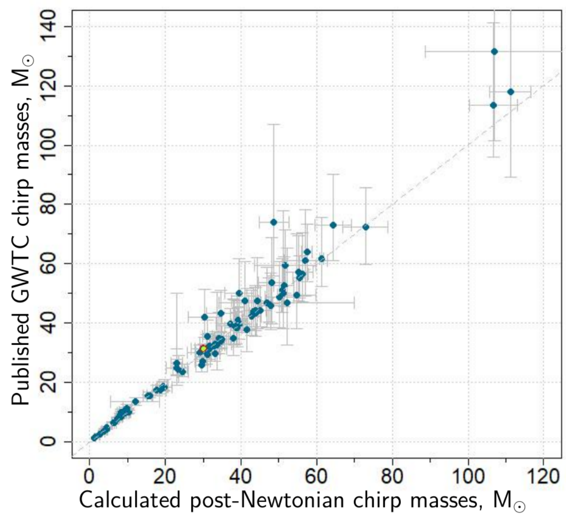

Fig. 1. Diagonal plot comparing the 97 GWTC chirp masses $\mathcal{M}_{\rm GWTC}$ (in the detector frame) and those calculated using the quadrupole post-Newtonian approximation: $\mathcal{M}_{\rm PN}$ (Eq. 1).

down. For example, for the last stable circular orbit in the Schwarzschild metric, the observed orbital frequency (and hence the frequency of GWs $f_{\rm GW} = 2f_{\rm orb}$) decreases by a factor of $\sqrt{2/3}$ (Zeldovich & Novikov 1967). Therefore, if the scheme for processing GW signals for curved spacetime gives the same result as the scheme for flat spacetime, then we can conclude that the time-dilation function $\alpha = \sqrt{g_{tt}}$ has somehow fallen out of the calculations for curved space.

A simple example given below for the case of the circular orbital motion of a test mass μ around the central mass M in the Schwarzschild metric shows how the time-dilation function α can, indeed, fall out of formalism even with an absolutely accurate relativistic calculation. For a circular orbit, the specific angular momentum $\tilde{\mathcal{L}} = p_\varphi/\mu = u_\varphi$ is

$$\tilde{\mathcal{L}} = \sqrt{\frac{rGM/c^2}{1 - 3GM/(rc^2)}}, \tag{2}$$

and the specific energy $\tilde{\mathcal{E}} = -p_t/\mu = -u_t$ is

$$\tilde{\mathcal{E}} = \frac{1 - r_g/r}{\sqrt{1 - 3GM/(rc^2)}}. \tag{3}$$

Consequently,

$$u^\varphi = \frac{d\varphi}{d\tau} = \frac{u_\varphi}{g_{\varphi\varphi}} = \frac{\tilde{\mathcal{L}}}{r^2}, \quad u^t = \frac{dt}{d\tau} = \frac{u_t}{g_{tt}} = \frac{\tilde{\mathcal{E}}}{1 - r_g/r}, \tag{4}$$

where τ is the proper (local) time in the source reference frame of the orbiter, g_{tt} and $g_{\varphi\varphi}$ are the spacetime metric coefficients for, respectively, time and the azimuthal angle in spherical coordinates, $\sqrt{g_{tt}}$ being the time-lapse function α. Then, by calculating $d\varphi/dt$ we obtain the angular frequency of the orbital motion in coordinate time t, i.e., in the reference system of a remote observer (detector):

$$\omega^{\mathrm{det}} = \frac{d\varphi/d\tau}{dt/d\tau} = \frac{u^{\varphi}}{u^{t}} = \sqrt{\frac{GM}{c^2 r^3}}, \tag{5}$$

which coincides with Kepler's non-relativistic formula for orbital motion. Thus, even though Eq. (5) is relativistically precise, the lapse and shift functions of the Schwarzschild metric fall out of the equations of motion when passing from the source reference frame (time τ) to the detector reference frame (time t) because they are canceled in the expression u^{φ}/u^{τ}.

The standard GW signal processing scheme uses a more complex coordinate transformation developed by Arnowitt et al. (1959) and abbreviated as the ADM formalism. Within this formalism, the energy-momentum tensor is integrated throughout space, after which the reduced action functional is varied, which gives equations of motion from which the lapse and shift functions are omitted, similar to Eq. (5). This mechanism of the omission of the lapse and shift functions is described in more detail in the reviews by Schäfer (1985); Schäfer & Jaranowski (2018).

4 Distance scaling

Due to the degeneracy of the relationship between the calculated chirp masses of merging binary systems and the distances to these systems, the theoretical amplitude of the GW signal is proportional to the 5/3 power of the chirp mass (Eq. 1). Therefore, if the chirp masses of GW sources are overestimated, then the estimates of distances to these sources are even more overestimated.

We can rescale these overestimated distances using the ratio between corrected and uncorrected chirp masses:

$$D_L^c = \left(\frac{\mathcal{M}^c}{\mathcal{M}}\right)^{5/3} D_L, \tag{6}$$

where D_L^c is the rescaled GW luminosity distance, D_L is the published distance, $\mathcal{M}$ and $\mathcal{M}^c$ are the original and corrected chirp masses of a coalescing binary system.

The recalculated distances D_L^c to 97 GW sources detected during the three observing runs of the LIGO–Virgo–Kagra collaboration are shown in Fig. 2 with black dots. For comparison, the original (uncorrected) chirp masses from the GWTC and their corresponding luminosity distances are shown as gray dots.

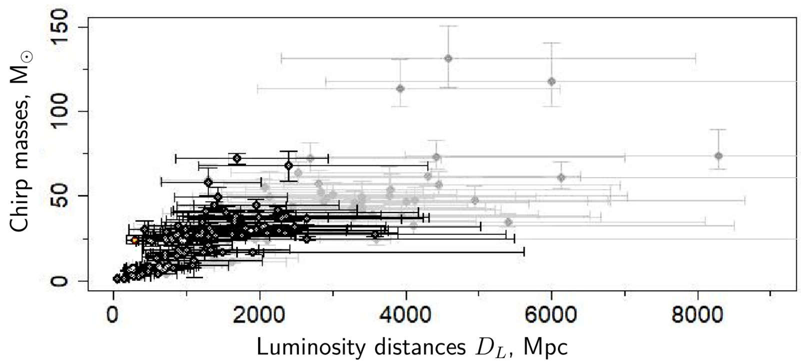

Fig. 2. Corrected chirp masses $\mathcal{M}^c$ of 97 black hole mergers (vetical axis) from three observing runs of the LIGO–Virgo–Kagra collaboration and distances D_L^c to these systems (abscissa) corrected by Eq. (6). The gray dots mark the initial (uncorrected) chirp masses and luminosity distances of the GW systems published in the GWTC.

5 Summary

It can be concluded that in the calculated chirp masses of binary black hole systems detected by interferometric gravitational wave detectors, there is a systematic overestimation effect. This effect is accompanied by an even greater overestimation of distances to the sources of gravitational waves.

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