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The radius of magnetosphere and the equilibrium radius are not the same!

N. Beskrovnaya[1] and N. Ikhsanov[1,2]

[1] Pulkovo Observatory, 65 Pulkovskoye shosse, Saint-Petersburg, 196140 Russia
[2] Institute of Applied Astronomy of the Russian Academy of Sciences, 10 Kutuzov Emb., Saint-Petersburg, 191187 Russia

Abstract. The process of accretion onto a neutron star with strong magnetic field is discussed. The equilibrium radius, which is defined by equating the pressure of the accreting material with the magnetic pressure due to the dipole magnetic field of the neutron star, is estimated for the cases of a spherical accretion flow and a Keplerian accretion disk. It is emphasized that the magnetospheric radius of an accreting star is defined by equating the mass accretion rate observed in the system with the rate of plasma diffusion into the magnetic field of the neutron star at the magnetospheric boundary. Following this definition we obtained a system of equations, which are the continuity equation and the pressure balance equation. We show that the radius of magnetosphere evaluated in this way significantly differs from the equilibrium radius. In particular, the Alfvén radius (which is just the equilibrium radius in a peculiar case of a spherically symmetric accretion flow) under the same conditions exceeds the magnetospheric radius of an accreting neutron star by more than an order of magnitude.

Keywords: accretion, accretion disks; pulsars: general; magnetic fields

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1 Introduction

We consider a Neutron Star (NS) with the mass $M_{\rm ns}$, the radius $R_{\rm ns}$, the surface magnetic field $B_{\rm ns}$ and the dipole magnetic moment $\mu = (1/2) B_{\rm ns} R_{\rm ns}^3$, which accretes matter at a rate $\dot{\mathfrak{M}}$. The questions are (i) at which distance (from the center of the NS) the balance between the pressure of the accretion flow and the pressure of the NS magnetic field can be reached (the equilibrium radius), and (ii) at which distance the rate of penetration of the accretion flow into the magnetic field of the NS reaches the rate of mass accretion $\dot{\mathfrak{M}}$, which is considered constant (a stationary accretion picture). We examine these questions for both the spherical and disk accretion scenarios within the corotation approximation, which implies that the angular velocity of matter at the inner radius of the accretion flow is equal to the angular velocity of the NS.

2 The equilibrium radius

The equilibrium radius, $r_{\rm eq}$, is defined by equating the magnetic pressure due to the dipole magnetic field of a NS, $p_{\rm m}(r) = \mu^2/2\pi r^6$, with the external pressure of the accretion flow, $p_{\rm acc}$.

The pressure of a spherical accretion flow is dominated by the dynamic (ram) pressure, which leads us to the equilibrium radius,

$$r_{\rm eq}^{\rm (sph)} = \left(\frac{\mu^2}{\dot{\mathfrak{M}} \sqrt{2GM_{\rm ns}}} \right)^{2/7} \simeq 1.2 \times 10^9 \, {\rm cm} \, \mu_{30}^{4/7} \, \dot{\mathfrak{M}}_{15}^{2/7} \, m^{1/7}, \qquad (1)$$

which is known as the Alfvén radius. Here $\mu_{30} = \mu/10^{30} \, {\rm G \, cm^3}$, $\dot{\mathfrak{M}}_{15} = \dot{\mathfrak{M}}/10^{15} \, {\rm g \, s^{-1}}$, and $m = M_{\rm ns}/1.4 \, {\rm M_\odot}$.

The pressure of an accretion disk in the corotation approximation is contributed mainly by the thermal gas pressure. The equilibrium radius in this case (Beskrovnaya & Ikhsanov 2024),

$$r_{\rm eq}^{\rm (d)} \simeq \frac{34 \, \alpha^{8/27} \, \mu^{8/27}}{\dot{\mathfrak{M}}^{7/27} \, (GM_*)^{7/27}} \simeq 3.6 \times 10^8 \, {\rm cm} \, \alpha^{8/27} \, \mu_{30}^{16/27} \, \dot{\mathfrak{M}}_{15}^{-7/27} \, m^{-7/27}, \qquad (2)$$

is smaller than the Alfvén radius under the same conditions. Here $\alpha = v_{\rm t} \ell_{\rm t}/c_{\rm s} h_0 \leq 1$ is the efficiency parameter which is used to normalize the velocity, $v_{\rm t}$ and scale, $\ell_{\rm t}$, of turbulent motions to the sound speed, $c_{\rm s}$, and half-thickness of the disk, h_0.

3 The magnetospheric radius

The magnetospheric radius of an accreting star is defined by equating the rate of plasma penetration into the magnetic field of the NS, $\dot{\mathfrak{M}}_{\rm in}$, to the mass accretion rate beyond the magnetospheric radius, $\dot{\mathfrak{M}}$. The penetration rate in the general case is limited as $\dot{\mathfrak{M}}_{\rm in} \geq \dot{\mathfrak{M}}_{\rm diff}$, where

$$\dot{\mathfrak{M}}_{\rm diff}(r) = S(r)\, v_{\perp}(r)\, \rho(r), \tag{3}$$

is the rate of plasma diffusion into the magnetic field of a NS at a given radius r. Here $S \simeq 4\pi r h_0$ is the area of interaction between the accretion disk and the NS magnetic field; $v_{\perp} \simeq \delta_{\rm m}/t_{\rm ff}$ is the velocity of plasma diffusion across the magnetic field lines, where $\delta_{\rm m} \simeq (D_{\rm eff} t_{\rm ff})^{1/2}$ is the thickness of the diffusion layer for the diffusion coefficient $D_{\rm eff}$ and $t_{\rm ff}$ is the free-fall time, which in the considered case represents the time of diffusion process; $\rho \simeq \mu^2/2\pi r^6 c_{\rm s}^2$ is the density of the accretion flow at the inner radius of the disk, which can be evaluated from the equation of pressure balance between the disk and the magnetosphere at its boundary.

Evaluating $\dot{\mathfrak{M}}_{\rm diff}(r)$ for the case of Bohm diffusion and solving equation $\dot{\mathfrak{M}}_{\rm diff}(r) = \dot{\mathfrak{M}}$ for r one finds $r = r_{\rm N}$, where (Beskrovnaya & Ikhsanov 2024)

$$r_{\rm N} \simeq \frac{0.16\,\lambda_0\,\mu^{6/11}}{\dot{\mathfrak{M}}^{4/11}\,(GM_{\rm ns})^{1/11}} \simeq 5.6 \times 10^7\,{\rm cm}\, \times\, \lambda_0\,\mu_{30}^{6/11}\,\dot{\mathfrak{M}}_{15}^{-4/11}\,m^{-1/11}, \tag{4}$$

and λ_0 is a dimensionless parameter of the order of unity. The radius $r_{\rm N}$ represents the minimum possible value of the magnetospheric radius of an accreting star. The inflowing matter cannot approach the NS at a closer distance since the rate of its diffusion into the stellar magnetic field at this radius reaches the rate of mass transfer in the disk. Furthermore, the radial velocity of plasma across the magnetic field lines at this radius is equal to the radial velocity of plasma in the disk.

4 Discussion

It is important to note that the physical problem of describing magnetospheric accretion essentially differs from the situation with the Earth magnetosphere. The solar wind in the latter case overflows the magnetic field of the Earth and only a tiny fraction of it penetrates into the magnetosphere. This occurs because of a high velocity of the solar wind which by a factor of 30–50 exceeds the parabolic velocity at the Earth's surface (so the Bondi radius proves to be much smaller than the radius of the Earth). The minimum size of the magnetosphere (a distance of the closest approach between the solar wind and the Earth) in this case can be roughly evaluated by equating the pressure of the dipole magnetic field of the Earth and the ram pressure of solar wind.

The situation with an accreting star is essentially different. The accretion flow is gravitationally bound with the accreting star (the Bondi radius is much larger than the magnetospheric radius). Almost all of the inflowing matter in this case is accreted onto the stellar surface. A stationary accretion picture under these circumstances can be constructed only if the rate of plasma penetration into the magnetosphere at its boundary is equal to the total mass accretion rate. Hence, the primary condition is the continuity equation and the pressure balance is a complementary (although a very important) condition which helps to reduce a number of free parameters of the model.

Historically, the magnetospheric radius of an accreting NS was calculated for the spherical accretion flow using the pressure balance equation with the Alfvén radius as a solution. But the rate of plasma diffusion into the magnetosphere in this case is too low to explain the observed luminosity of X-ray pulsars. To solve this problem an assumption about instabilities of the magnetospheric boundary had been invoked (for discussion see, e.g., Ikhsanov & Pustil'nik 1996, and references therein).

The equilibrium magnetospheric boundary of a NS accreting from a spherical flow under the condition $T(r_{\rm A}) = T_{\rm ff}(r_{\rm A})$ is interchange stable (Arons & Lea 1976). Here $T(r_{\rm A})$ and $T_{\rm ff}(r_{\rm A})$ are the gas and the free-fall temperature at the magnetospheric boundary. The boundary could be unstable under the condition $T(r_{\rm A}) \ll T_{\rm ff}(r_{\rm A})$. But the equilibrium shape of the magnetosphere for this case has been never investigated. Instead, the stability analysis was performed extrapolating the solution for the case of $T(r_{\rm A}) = T_{\rm ff}(r_{\rm A})$ to the case of $T(r_{\rm A}) \ll T_{\rm ff}(r_{\rm A})$. A validity of this extrapolation is rather questionable. Furthermore, the matter in this situation tends to overflow the magnetospheric boundary towards the magnetic pole regions and to accumulate in the magnetic cusps, which are interchange stable (Michel 1977). It therefore appears that the continuity equation for the case of spherical accretion has not been solved so far. The diffusion-driven accretion scenario is currently available for the case of disk accretion in which, as shown above, the magnetospheric radius significantly differs from the traditionally invoked Alfvén radius, which is just the equilibrium radius in the case of a NS undergoing spherical accretion.

References

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