



# Estimation of the dynamic masses of open star clusters

M. Kulesh and A. Seleznev

Ural Federal University, 51 Lenin Str., Ekaterinburg, 620000 Russia

**Abstract.** The determination of star cluster masses is crucial for the study of cluster dynamics, the evaluation of gravitational binding, and the assessment of star formation efficiency. Traditional photometric approaches face challenges such as incompleteness, the problem of evaluating the uncertainty in the mass-luminosity relation, contamination by binaries, and so on. Dynamic (virial) approaches, which provide an estimate of the total cluster mass, have a difficulty in estimating the velocity dispersion. This study proposes a method to calculate the dynamic masses by using proper motions, taking into account for their observational errors. Using modern data for open clusters and samples of their probable members, we estimate dynamic masses of 833 open clusters. The resulting dynamic mass distribution is nearly log-normal with the mode of  $2400 M_{\odot}$ .

**Keywords:** open clusters and associations: general; stars: kinematics and dynamics; methods: statistical

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# 1 Introduction

The determination of star cluster masses is critical for a variety of astrophysical applications, including the study of cluster dynamics, the evaluation of gravitational binding, and the assessment of star formation efficiency in young clusters. Traditional methods for estimating cluster masses include photometric and dynamic (or virial) approaches, each of which has its own challenges.

Photometric mass estimation relies on the cluster member list or luminosity function and a mass-luminosity relation (see for example Seleznev et al. 2017). This method, which uses theoretical isochrones, faces difficulties in evaluating the uncertainty of the mass-luminosity relation. To mitigate this, one can use isochrone tables from different authors or analytical expressions for the mass-luminosity relation (Eker et al. 2015 as an example). However, the photometric mass represents only a lower limit of the real cluster mass due to uncertainties at the lower end of the luminosity function, invisible stellar remnants, and unresolved binaries (Seleznev 2016; Borodina et al. 2021).

Dynamic mass estimation, on the other hand, depends on the velocity dispersion of the cluster stars. While this method takes all cluster stars into account, it requires an accurate determination of the velocity dispersion and the structural properties of the cluster. We use both a simple “virial” formula, and the formula that takes into account the non-stationarity of the cluster and the influence of the Galactic gravitational field (Danilov & Loktin 2015). We determine the necessary structural properties using a numerical model. In addition, when one uses the radial velocities, a contamination by single-line spectroscopic binaries can inflate the velocity dispersion, leading to an overestimation of the dynamic mass. These issues are discussed in more detail in Seleznev et al. (2017) and Kulesh et al. (2024).

Modern catalogs of star clusters include the necessary parameters necessary to estimate the dynamic masses, such as the samples of probable member stars, proper motions and their errors, distance estimates to the clusters, galactic coordinates of their centers, and estimates of the King surface density profile parameters  $r_t$  and  $r_c$  (King 1962). We use the Hunt & Reffert (2023) catalog based on the Gaia DR3 (Gaia Collaboration et al. 2023) and calculate the virial and dynamic masses from the velocity dispersion of the Gaia DR3 proper motions. In addition, this work proposes a method to account for the observational errors of the proper motions in the final velocity dispersion.

## 2 Data and processing

We obtained the list of clusters and their parameters from Hunt & Reffert (2023). Then we cut the list to the clusters presented in Dias et al. (2021). As a result, we get a list of 1410 clusters.

For each cluster in the list, a sample of stars with a membership probability greater than 0.5 and a Renormalised Unit Weight Error (RUWE)  $\leq 1.4$  has been selected. We use the selection by RUWE to exclude probable multiple stars. Then we use the 3-sigma rule for the proper motion distributions  $\mu_\alpha$  and  $\mu_\delta$  for each cluster sample to remove the outliers.

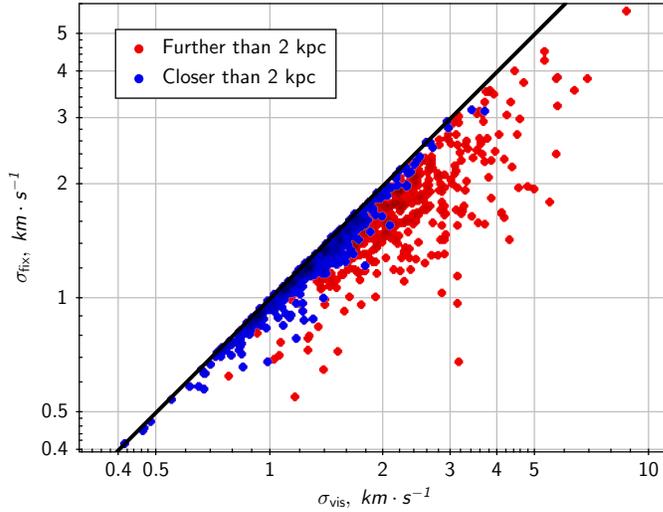
We then checked the normality of the proper motion distributions using the omnibus test of normality (D’Agostino 1971). After removing the clusters with p-value less than 0.05, 873 clusters remained.

We convert the proper motions  $\mu_\alpha$  and  $\mu_\delta$  and their errors  $e_{\mu_\alpha}$  and  $e_{\mu_\delta}$  into tangential velocities  $v_\alpha$  and  $v_\delta$  and their errors  $e_{v_\alpha}$  and  $e_{v_\delta}$ , using the median distance estimate to the cluster (Hunt & Reffert 2023), hereafter referred to as  $dist_0$ . These errors broaden the velocity distribution. We assume that both the velocity and its error distributions are Gaussian, as in our previous work (see Kulesh et al. 2024 for details). Let the dispersions of these distributions be  $\sigma_{\text{vis}|\alpha,\delta}^2$  and  $\sigma_{\text{err}|\alpha,\delta}^2$ , respectively. Then the corrected dispersion is:

$$\sigma_{\text{fix}|\alpha,\delta}^2 = \sigma_{\text{vis}|\alpha,\delta}^2 - \sigma_{\text{err}|\alpha,\delta}^2. \quad (1)$$

To estimate the velocity dispersion, we modify the method of Kulesh et al. (2024) and do not use the kernel density estimation for distributions and their analytic approximations, since these procedures introduce additional errors. The dispersion  $\sigma_{\text{vis}|\alpha,\delta}^2$  can be estimated simply as the dispersion of the sample, as in the case of the maximum likelihood method. We estimate the dispersion of the errors  $\sigma_{\text{err}|\alpha,\delta}^2$  as the square of the median of the error distributions  $e_{v_\alpha}$  and  $e_{v_\delta}$ , respectively. Figure 1 shows the difference between the corrected and uncorrected standard deviations of the total velocity (see below). For 40 clusters, their  $\sigma_{\text{err}|\alpha,\delta} > \sigma_{\text{vis}|\alpha,\delta}$ . In this case, eq. 1 is obviously incorrect, and we remove these clusters. The final list contains 833 clusters.

Next, we follow to the procedure described in Kulesh et al. (2024) (Section 5). We use the velocity dispersion in the tangential direction to estimate the dynamical mass. We assume that the velocity distribution in the line-of-sight direction is the same as the velocity distribution in both tangential directions, and evaluate the velocity dispersion to be  $\sigma^2 = (3/2)(\sigma_\alpha^2 + \sigma_\delta^2)$ .



**Fig. 1.** The difference between the corrected  $\sigma_{\text{fix}}$  and uncorrected  $\sigma_{\text{vis}}$  standard deviations of the total velocity. Effect is more dramatic for distant clusters

We estimate the virial cluster mass using the formula:

$$M_{\text{vir}} = \frac{2\sigma^2 \bar{R}}{G}, \quad (2)$$

where  $\bar{R} = \langle 1/r_{ij} \rangle^{-1}$  is the mean inverse star-to-star distance,  $G$  is the gravitational constant. We estimate the cluster's dynamic mass using the formula from Danilov & Loktin (2015). This formula takes into account the gravitational field of the Galaxy and the non-stationarity of the cluster:

$$M_{\text{dyn}} = \frac{2R_u \bar{R}}{G (\bar{R} + R_u)} \left( 2\sigma^2 - \frac{\alpha_1 + \alpha_3}{3} \langle r^2 \rangle \right), \quad (3)$$

where  $R_u = \langle 1/r_i \rangle^{-1}$  is the mean inverse star distance to the cluster centre,  $\langle r^2 \rangle$  is the mean square of the star distance to the cluster centre,  $\alpha_1$  and  $\alpha_3$  are the field constants characterising the Galactic potential in the vicinity of the cluster (Chandrasekhar 1942). The values of  $\alpha_1$  and  $\alpha_3$  have been calculated by adopting the Galactic potential model of Kutuzov & Osipkov (1980), using the known galactic coordinates of the cluster  $l_0$  and  $b_0$ , together with the known distance to the cluster  $dist_0$  and the distance from the Galactic centre to the Sun  $R_\odot = 8200$  pc.

The estimates of the King profile parameters  $r_c$  and  $r_t$  given in Hunt & Reffert (2023) allow us to immediately obtain the analytical form of the probability distribution of stars in the cluster as a function of distance from the cluster centre, following

to the spatial density profile derived in King (1962):

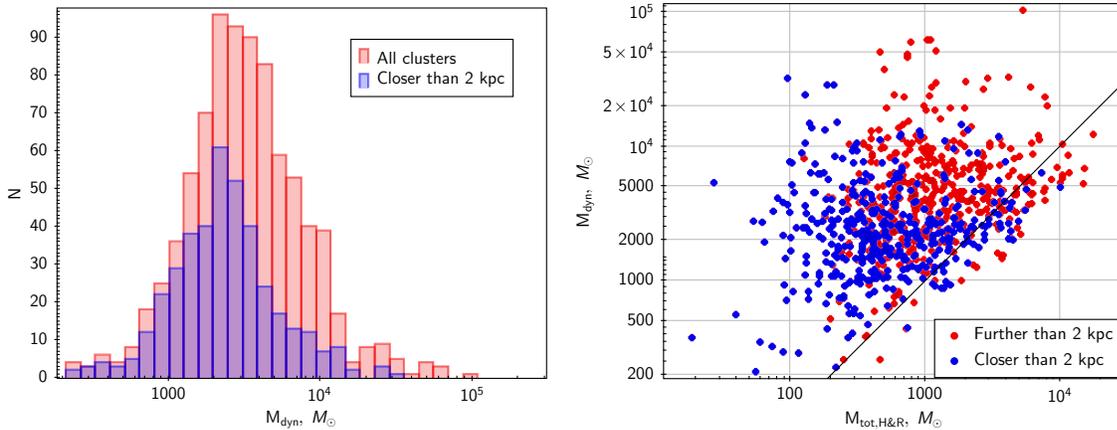
$$\rho(r) \sim \left(\frac{r}{z}\right)^2 \left(\frac{\arccos z}{z} - \sqrt{1 - z^2}\right), \quad z = \sqrt{\frac{r_c^2 + r^2}{r_c^2 + r_t^2}}.$$

This allows for simple Monte Carlo modeling via rejection-acceptance sampling and estimation of  $R_u$ ,  $\bar{R}$  and  $\langle r^2 \rangle$ . The modeling was run 20 times for each sample to estimate the spread of the parameters.

Finally, the resulting virial and dynamic masses were calculated using eq. 2 and eq. 3 with the corrected  $\sigma_{\text{fix}}^2$  dispersion of the total velocity.

### 3 Results

The resulting distribution of  $M_{\text{dyn}}$  for all clusters in the list is shown in Fig. 2, left. This distribution is almost log-normal with the mode at about  $2400 M_{\odot}$ .



**Fig. 2.** (Left): Dynamic mass distribution of open clusters is nearly log-normal (with p-value  $< 0.01$  for clusters closer than 2 kpc) with mode at  $2400 M_{\odot}$  (Right): Relation of  $M_{\text{dyn}}$  to  $M_{\text{tot}}$  of the same cluster from Hunt & Reffert (2024); correlation is weak (0.3) and the dynamic masses are systematically greater

We compare our results with the final version of the open cluster catalogue of Hunt & Reffert (2024) (see Fig. 2, right). Their mass distribution is also log-normal too, but with the mode of about  $340 M_{\odot}$ .

$M_{\text{tot}}$  from Hunt & Reffert (2024) are systematically less than  $M_{\text{dyn}}$ . We intend to further analyze this difference in detail.

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