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Velocities-distance optimization of the rotation model of a homogeneous flat subsystem of Galactic objects: masers

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Abstract. A statistically correct method for optimizing the parameters of the kinematic model for a homogeneous set of Galactic objects has been developed and implemented, which includes minimizing the squares of relative deviations from the observed radial velocity, proper motions, and distant characteristic. The latter refers to the trigonometric parallax (in the case of absolute distances) or the distance modulus (in the case of relative, i.e., photometric, distances). The solution lies in the technique of the principle of maximum likelihood. The presence of measurement errors and natural (dynamic, astrophysical) velocity dispersion is taken into account; the parameters of the latter (velocity ellipsoid) are estimated. The final iterative algorithm includes optimization of the smoothness of the rotation law and a flexible procedure for eliminating outliers in the data, generalized to a four-dimensional field of residuals. The method, without requiring restrictions on the magnitude of random errors in the distant characteristic, directly takes into account the non-Gaussian distribution of errors in heliocentric distances, thereby correcting the corresponding systematic shifts in the parameters of the model and the average rotation curve of the subsystem under consideration. The inclusion of distance uncertainties in the probabilistic model strongly affects the estimates of natural velocity variances (reduces them), and also generally reduces the variances of model parameters. Applying the method to HMSFR maser sources with trigonometric parallaxes gives the following parameter estimates: distance to the center of the Galaxy $R_0 = 7.88 \pm 0.12$ kpc, angular velocity of rotation of the maser subsystem on the solar circle $\omega_0 = 28.43 \pm 0.22$ $\mathrm{km\,s^{-1}\,kpc^{-1}}$, corresponding linear velocity $\theta_0 = 224 \pm 4$ $\mathrm{km\,s^{-1}}$, the angular velocity of the Sun's rotation around the center of the Galaxy $\omega_\odot = 30.40 \pm 0.20$ $\mathrm{km\,s^{-1}\,kpc^{-1}}$.

Keywords: methods: data analysis; Galaxy: fundamental parameters, kinematics and dynamics

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1 Introduction

Studies of the kinematics of the Milky Way on a large scale based on data on ensembles of Galactic objects require measurements not only of the components of the velocities of objects, but also of the heliocentric distances to them. However, with appropriate kinematic modeling, the traditional difficulty is taking into account the uncertainty of heliocentric distances. Failure to account for such uncertainty can lead to significant systematic errors in the results. This problem became especially relevant after the appearance of mass joint determinations of proper motions and trigonometric parallaxes – e.g., catalogs of maser sources (Reid et al. 2019; VERA Collaboration et al. 2020), the *Gaia* catalog (Gaia Collaboration et al. 2023) – because objects with large linear and relative errors in distances are guaranteed to be present in such databases.

In early work, Reid et al. (2009) attempted to account for the uncertainty of the parallaxes in the framework of the method of least squares, evaluating model parallax by radial velocity, but then, faced with difficulties, they abandoned this approach and returned to the usual "velocity-only" fitting (Reid et al. 2014, 2019). However, the uncertainty of the distance for many masers turns out to be too large to be ignored. At the same time, the understanding is gradually spreading that the maximum likelihood method (MLM) is a promising tool for such research. Rastorguev et al. (2017) take into account the uncertainty of distances within the framework of the MLM by including partial derivatives of the first order in distance for velocities in the covariance matrix. However, such accounting ignores the non-Gaussian distribution of errors in distances and is suitable only for small uncertainties of the latter.

A better idea is the individual reduction of heliocentric distances by including distributions of distant characteristics of objects in the likelihood function. Pont et al. (1994) applied this approach to photometric distances, using only radial velocities, and while fixing the parameters of the velocity ellipsoid. In the case of involving proper motions and parallaxes, the same approach was implemented within the MLM by Aghajani & Lindegren (2013) and Ding et al. (2013). However, the analytical solution given by these authors for the reduced (corrected) parallaxes is approximate and can only be applied with small parallax errors.

In this paper, we propose a spatial-kinematic modeling method based on the principle of maximum likelihood, which works with a minimum of assumptions, takes into account natural and measurement variances, does not require limits on parallax errors and directly takes into account the non-Gaussian error distribution in heliocentric distances. The method is applied to HMSFR masers.

2 Data

From maser sources, the HMSFR class masers are of the greatest interest for studying the kinematics of the Galaxy as a very "cold" subsystem of a thin disk with absolute distances, moreover, characterized by small parallax uncertainties even for large heliocentric distances (see Nikiforov & Veselova 2018). To obtain a sample of HMSFR measurements, we used the catalogues in Reid et al. (2019); VERA Collaboration et al. (2020), supplementing them with new data from the literature: Xu et al. (2021); Sakai et al. (2021); Bian et al. (2022); Mai et al. (2023); Hyland et al. (2023, 2024). The full sample includes 210 HMSFR masers.

3 Method

A variable, i.e., depending on unknown parameters of the model, part of the logarithmic likelihood function has the following form:

$$\begin{aligned}\mathcal{L}(\mathbf{a}) = \;& \sum_{j=1}^{N}\left[\ln(\sigma_{V_r})_j + \ln(\sigma_{\mu_l'})_j + \ln(\sigma_{\mu_b})_j\right] + \frac{1}{2}\sum_{j=1}^{N}\min_{\varpi_{0,j}}\left\{\frac{[V_{r,j} - V_{r,\mathrm{mod}}(\varpi_{0,j})]^2}{(\sigma_{V_r})_j^2} + \right.\\ & \left. + \frac{[\mu_{l,j}' - \mu_{l,\mathrm{mod}}'(\varpi_{0,j})]^2}{(\sigma_{\mu_l'})_j^2} + \frac{[\mu_{b,j} - \mu_{b,\mathrm{mod}}(\varpi_{0,j})]^2}{(\sigma_{\mu_b})_j^2} + \frac{(\varpi_j - \varpi_{0,j})^2}{\sigma_{\varpi,j}^2}\right\}.\end{aligned} \quad (1)$$

Here $\mathbf{a}$ is the vector of parameters; $V_{r,j}$, $\mu_{l,j}'$, $\mu_{b,j}$, and ϖ_j are catalog (measured) values of heliocentric radial velocity V_r, longitude ($\mu_l' \equiv \mu_l \cos b$) and latitude ($\mu_b$) proper motions, and parallax ϖ for the jth object; $\varpi_{0,j}$ — the reduced parallax value of the jth object; N is the sample size. The total variances of the velocity characteristics for the jth object are the the sums of two components:

$$(\sigma_{V_r})_j^2 = (\tilde{\sigma}_{V_r})_j^2 + (\sigma_{V_r}^*)_j^2, \quad (\sigma_{\mu_l'})_j^2 = (\tilde{\sigma}_{\mu_l'})_j^2 + (\sigma_{\mu_l'}^*)_j^2, \quad (\sigma_{\mu_b})_j^2 = (\tilde{\sigma}_{\mu_b})_j^2 + (\sigma_{\mu_b}^*)_j^2, \quad (2)$$

where $\tilde{\sigma}_j^2$ are the measuring variance, $\sigma_j^{*2} = \sigma_j^{*2}(l_j, b_j, \varpi_{0,j}; \sigma_R, \sigma_\theta, \sigma_Z, R_0)$ are contributions of natural dispersion, i.e., the velocity ellipsoid $(\sigma_R, \sigma_\theta, \sigma_Z)$. Here l_j and b_j are the longitude and latitude of jth object, correspondingly; R_0 is the distance from the Sun to the center of the Galaxy. The reduced parallax is considered as a random variable distributed according to the normal law $\mathrm{N}(\varpi_j, \sigma_{\varpi,j}^2)$, where $\sigma_{\varpi,j}$ is the catalog uncertainty of parallax. The triple of values $(l_j, b_j, \varpi_{0,j})$ determines the point of the (non-orthogonal) projection of the object onto the kinematic model, taking into account all random factors.

The following model was used for velocity characteristics:

$$
\begin{aligned}
V_{r,\mathrm{mod}} &= T_n(\varDelta R)\,\frac{R_0}{R}\sin l\cos b - u_\odot\cos l\cos b - v_\odot\sin l\cos b - w_\odot\sin b,\\
k\mu'_{l,\mathrm{mod}} &= T_n(\varDelta R)\left(\frac{R_0\cos l}{r}-\cos b\right)R^{-1} - \omega_0\cos b + (u_\odot\sin l - v_\odot\cos l)/r, \qquad (3)\\
k\mu_{b,\mathrm{mod}} &= T_n(\varDelta R)\,\frac{R_0}{Rr}\sin l\sin b + (u_\odot\cos l\sin b + v_\odot\sin l\sin b - w_\odot\cos b)/r.
\end{aligned}
$$

Here $T_n(\varDelta R) \equiv -2A\varDelta R + \sum_{i=2}^{n}\frac{\theta_i}{i!}(\varDelta R)^i$ is the contribution of the expansion of the average *linear* rotational velocity $\theta(R)$ of the subsystem in a series of $\varDelta R \equiv R - R_0$, $n \geq 1$; A is the Oort parameter; $\theta_i \equiv \left.\frac{d^i\theta}{dR^i}\right|_{R=R_0}$; $R = R(r, l, b)$ is the Galactoaxial distance, $r = 1/\varpi$ is the heliocentric distance; ω_0 is the angular velocity of rotation of the subsystem at R_0; $u_\odot$, $v_\odot$, $w_\odot$ are components of the *residual* velocity of the Sun relative to the rest standard of the subsystem; $k = 4.7406$.

The parameter vector $\mathbf{a}$ is found by minimizing the function $\mathcal{L}(\mathbf{a})$ for the selected value of the model order n. For a fixed sample of objects, a solution was sought for n from 1 to 8. The optimal order of the n_o model was determined based on the behavior of dispersion in the Galactic plane, $\sigma^2_\mathrm{plane} \equiv \sigma^2_R + \sigma^2_\theta$, depending on n using a partial F-test (see details in Nikiforov 1999a,b). For optimal order, an outlier exclusion procedure was performed using a one-dimensional algorithm (Nikiforov 2012) and its generalization to a four-dimensional field of residuals based on statistics χ^2_4. The value of the algorithm parameter $L' = 3$ was used to estimate the parameters of the velocity ellipsoid; with $L' = 1$ (a more strong criterion), the remaining parameters were estimated (see details in Nikiforov 2012). After each series of exceptions, solutions were searched anew for different orders of n and the whole procedure was repeated from the beginning until no more exceptions were needed.

4 Results

In all the considered cases, the optimal model was of the order $n_\mathrm{o} = 3$. For this order, the results are presented in Table 1. They show that not taking into account the uncertainty of parallaxes (conventional 3D fitting) leads to a noticeable shift in the estimates of some parameters compared to our proposed method (4D fitting). In particular, the 3D estimate of R_0 is overestimated by $\sim$0.4 kpc; respectively, the estimate of the Oort parameter A is underestimated. The natural dispersions of σ_R and σ_θ with 3D fitting are overestimated by a multiple. At the same time, the non-zero parameters of the velocity ellipsoid, even after excluding outliers within the 4D fitting, show that the residuals cannot be fully explained only by measurement errors and the natural velocity dispersion is quite measurable for the HMSFR subsystem. It should be emphasized that estimates of R_0 based on these data for 4D fittings are very stable to various calculation options. The final results in Table 1 are highlighted in bold. Figure 1, representing the rotation curve based on HMSFR data and the resulting model, also demonstrates that parallax errors should not be ignored.

Table 1. Results of spatial-kinematic modeling based on HMSFR maser data for a model of optimal order $n_o = 3$

N	DoM	Exclusions	R_0 (kpc)	ω_0 $\left(\frac{\text{km/s}}{\text{kpc}}\right)$	A $\left(\frac{\text{km/s}}{\text{kpc}}\right)$	$u_\odot$ $\left(\frac{\text{km}}{\text{s}}\right)$	$v_\odot$ $\left(\frac{\text{km}}{\text{s}}\right)$	$w_\odot$ $\left(\frac{\text{km}}{\text{s}}\right)$	σ_R $\left(\frac{\text{km}}{\text{s}}\right)$	σ_θ $\left(\frac{\text{km}}{\text{s}}\right)$	σ_z $\left(\frac{\text{km}}{\text{s}}\right)$	θ_2 $\left(\frac{\text{km/s}}{\text{kpc}^2}\right)$	θ_3 $\left(\frac{\text{km/s}}{\text{kpc}^3}\right)$
210	3D	—	8.317	28.43	14.82	3.47	16.41	8.27	20.36	15.79	4.80	−1.83	1.16
			$^{+0.140}_{-0.126}$	±0.39	±0.41	±1.71	$^{+1.92}_{-1.91}$	±0.57	$^{+1.49}_{-1.23}$	$^{+1.49}_{-1.38}$	$^{+0.66}_{-0.64}$	±0.32	±0.20
210	4D	—	7.935	28.3	15.77	6.21	16.0	8.36	11.06	4.8	4.10	−1.29	1.30
			$^{+0.123}_{-0.120}$	±0.2	$^{+0.26}_{-0.25}$	$^{+1.04}_{-1.07}$	±1.0	±0.54	$^{+0.73}_{-0.83}$	±0.9	$^{+0.65}_{-0.64}$	$^{+0.22}_{-0.23}$	$^{+0.18}_{-0.17}$
201	4D	$L' = 3$	7.897	28.46	15.56	6.43	15.67	8.32	**7.13**	**4.64**	**3.45**	−1.30	1.09
			$^{+0.116}_{-0.113}$	±0.22	±0.22	±0.85	$^{+0.93}_{-0.94}$	$^{+0.52}_{-0.53}$	$^{+0.83}_{-0.77}$	$^{+0.84}_{-0.85}$	$^{+0.64}_{-0.65}$	$^{+0.20}_{-0.21}$	$^{+0.14}_{-0.13}$
197	4D	$L' = 1$	**7.881**	**28.43**	**15.46**	**6.09**	**15.55**	**8.22**	(7.13)	(4.64)	(3.45)	**−1.34**	**1.03**
			$^{+0.119}_{-0.115}$	±0.22	±0.23	±0.86	±0.92	±0.52				±0.20	±0.14

The second column "DoM" indicates the dimension of the method: "3D" means that catalog values of parallaxes were used (parallax errors were not taken into account); "4D" means that the reduced parallaxes were found (parallax errors were taken into account).

5 Summary

The proposed method (4D fitting) makes it possible to eliminate systematic errors in the results of kinematic modeling due to the uncertainty of parallaxes. As a result of its application to HMSFR masers, new estimates of fundamental parameters were obtained, in particular, $R_0 = 7.88 \pm 0.12$ kpc, $\omega_0 = 28.43 \pm 0.22$ km s^{-1} kpc^{-1}. The corresponding derived parameters: the linear velocity of rotation of the HMSFRs at $R = R_0$ is $\theta_0 = 224 \pm 4$ km s^{-1}, the angular velocity of the Sun's rotation around the center of the Galaxy is $\omega_\odot = 30.40 \pm 0.20$ km s^{-1} kpc^{-1}, $\theta_\odot \equiv \omega_\odot R_0 = 240 \pm 4$ km s^{-1}.

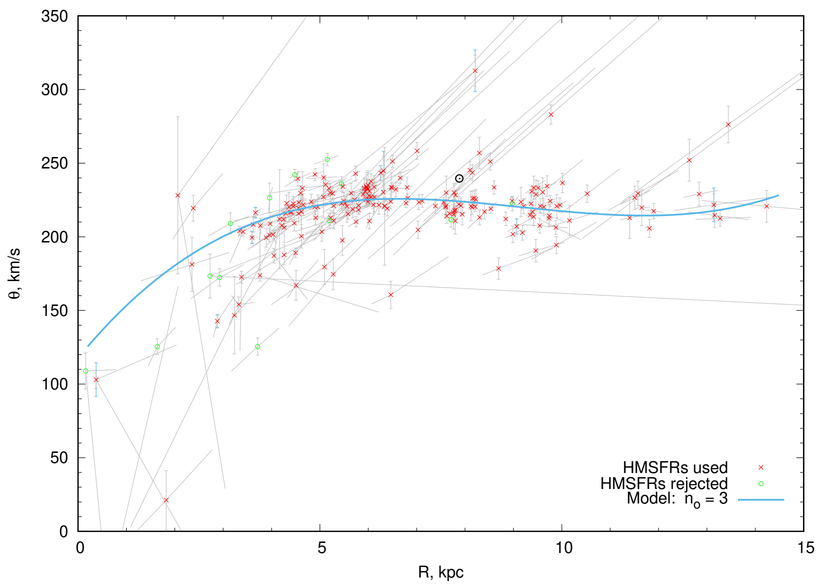

Fig. 1. The rotation curve for the optimal order model ($n_o = 3$) according to the HMSFR maser data. Vertical bars display errors in measuring the velocity characteristics of objects. Inclined ("broken") bars show a change in the position of an object on the rotation curve when the parallax changes by the amount of its uncertainty in both directions. The adjusted uncertainties of proper motions for some objects are shown as additional blue bars. The Sun sign corresponds to the point $(R_0, \theta_\odot)$.

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