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Random force distribution for fractal stellar medium in the solar neighborhood based on Gaia DR2 data

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Abstract. We examine the law of the distribution of the magnitude of random force in the nearest-neighbor approximation (the generalized Holtsmark distribution) and asymptotics of the Holtsmark distribution for large random forces for the fractal stellar medium in the Solar neighborhood with fractal dimension $D \approx 2.41$. Our investigation is based on the study of 200 000 stars of all spectral types at a distance of 1 to 100 pc from the Sun. We demonstrate that for fractal stellar medium slower decreasing random force distribution takes place. It indicates an important role of strong fields in the kinetics of gravitating medium in comparison with a classical uniform stellar medium.

Keywords: Galaxy: kinematics and dynamics, solar neighborhood, stellar content

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1 Introduction

The fractal structure of the environment of galaxies was predicted by Carpenter (1938) and de Vaucouleurs (1970). They found, that the number density n of galaxies in cluster decreases with the growth of their characteristic sizes according to the power law, $n(r) \sim r^{-1.7}$, where r – the size of a cluster, n – density of galaxies in cluster. Mandelbrot (1977) interpreted this law as a special case of stochastic self-similarity of three-dimensional random fractal sets for which the following relation holds:

$$n(r) \sim r^{-\alpha} \ , \tag{1}$$

where r – the characteristic size of the increasing volume, $n(r)$ – mean star conditional density, α – the exponent. Mandelbrot applied fractal dimension for fractal stellar media and showed that the fractal dimension is equal to $D = 3 - \alpha$ and $0 \leq D \leq 3$.

Recently, we have studied the spatial distribution of 200 000 stars of all spectral types in a solar neighborhood at a distance of 1 to 100 parsecs from the Sun from the observational data of the Gaia (DR2, 2018). The analysis of the spatial distribution of stars is carried out by the "mass-radius" method. Our calculations have shown that the mean conditional stellar density $n(r)$ is approximated by power laws of the form:

$$n(r) = h\, r^{-\alpha} \ , \tag{2}$$

where $h = 1.654$ and $\alpha = 0.586$, with the Pearson significance level $R^2 \approx 0,992$. It has confirmed the law of de Vaucouleurs–Mandelbrot for fractal structures in a gravitating media. Thus, the fractal dimension is $D = 3 - \alpha \approx 2.41$.

2 Distribution of the magnitude of random force in the fractal medium

Chumak & Rastorguev (2016) has derived an exact solution for the distribution of the magnitude of the fluctuating random force in the case of homogeneous space distribution of stars. This solution has the form of the Holtsmark distribution:

$$W(|\mathbf{F}|) = H(\beta) a^{2/3} \ , \tag{3}$$

where β is a dimensionless coefficient, and the distribution of relative forces $H(\beta)$ looks like

$$H(\beta) = \frac{2}{\pi\beta} \int\limits_0^\infty \exp\left[-(x/\beta)^{3/2}\right] x \sin x dx \ . \tag{4}$$

In the case of equal-mass stars, $a = (4/15)(2\pi Gm)^{3/2}n$, and $\beta = |\mathbf{F}|/a^{2/3}$.

Chandrasekhar (1943) has shown that the asymptotics of the Holtsmark distribution in the approximation of large random forces exactly coincides with the distribution of the random force acting on a unit-mass test star. We get for the nearest neighbor of mass m located at a distance r: $|\mathbf{F}| = Gm/r^2$.

The asymptotic formula for $W(F)$ can be derived from the distribution of the distance to the nearest neighbor $w(r)$,

$$w(r)dr = 4\pi \exp(-4\pi r^3 n/3)nr^2 dr \ , \tag{5}$$

that results in

$$W(|\mathbf{F}|)d|\mathbf{F}| = 4\pi n(Gm)^{3/2} \exp\left[-(4\pi n/3)(Gm)^{3/2}|\mathbf{F}|^{-3/2}\right] |\mathbf{F}|^{-5/2} d|\mathbf{F}| \ . \tag{6}$$

A generalization of the Holtsmark distribution was obtained for the fractal space star density (Chumak & Rastorguev 2016) assuming that the force acting onto the test star is also determined by its nearest neighbor exclusively and taking into account the fractal structure of the stellar medium. To this end, the distribution of the distance to the nearest neighbor should be derived using eq. (2):

$$w(r)dr = 4\pi h \exp\left[-(4\pi h/D)\, r^D\right] r^{D-1} dr \ . \tag{7}$$

This is an analog of formula (5), i.e., the generalization of the law of the nearest-neighbor distribution for the case of fractal distribution. Further, taking into account (7), it was derived the generalization of formula (6) for the case of power-law density distribution (2):

$$W(|\mathbf{F}|D)d|\mathbf{F}| = 4\pi h(Gm)^{D/2} \exp\left[-(4\pi h/D)(Gm/|\mathbf{F}|)^{D/2}|\right] \mathbf{F}|^{-(D+2)/2} d|\mathbf{F}| \ . \tag{8}$$

Let us define the dimensionless argument of the exponential function in (8), $x = (4\pi h/D)(Gm/F)^{D/2}$. Then eq. (8) can be rewritten as

$$AW(F|D) = (3x)^{(D+2)/2} e^{-x} \ , \tag{9}$$

where $A = (4\pi h)^{2/D}/(Gm)$.

Figure 1 shows the standard Holtsmark distribution (the lower curve) and our derived distribution (the upper curve) which takes into account the fractal nature of the 200 000 local stars with dimension $D \approx 2.41$. It is evident from (9) that the probability distribution $W(F)$ decreases most rapidly for the uniform star distribution, when $D = 3$.

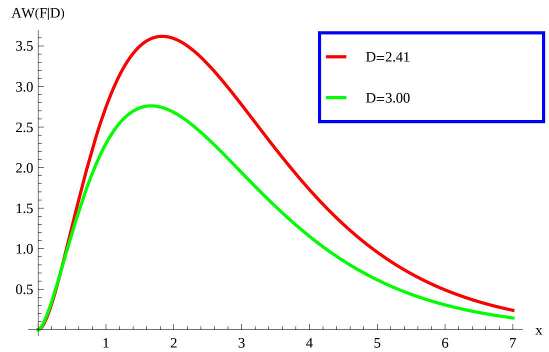

Fig. 1. The distribution $W(F|D)$ ($D \approx 2.41$, the upper curve) and the standard Holtsmark distribution ($D = 3$, the lower curve).

3 Summary

Analysis of space distribution of 200 000 stars of all spectral types at a distance of 1 to 100 pc from the Sun based on Gaia DR2 data indicates the presence of fractal structures with fractal dimension $D \approx 2.41$. The study of probability distribution of magnitude of random force in the nearest-neighbor approximation demonstrate large role of strong fields and therefore paired encounters of stars in collisional kinetics of gravitating medium in comparison with classical uniform stellar medium.

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