



# Methods for analyzing the light curves of the non-thermal radiation in active galactic nuclei

M. Khabibullina<sup>1</sup>, V. Stolyarov<sup>1,2</sup>, A. Mikhailov<sup>1</sup>, D. Kudryavtsev<sup>1</sup>, V. Vlasyuk<sup>1</sup>,  
Yu. Sotnikova<sup>1,3</sup>, T. Mufakharov<sup>1,3</sup>, and D. Morozova<sup>4</sup>

<sup>1</sup> Special Astrophysical Observatory of the Russian Academy of Sciences,  
Nizhny Arkhyz, 369167 Russia

<sup>2</sup> Astrophysics Group, Cavendish Laboratory, University of Cambridge, Cambridge, CB3 0HE UK

<sup>3</sup> Kazan Federal University, 18 Kremlyovskaya St, Kazan, 420008 Russia

<sup>4</sup> Saint Petersburg State University, St. Petersburg, 199034 Russia

**Abstract.** We present different methods for analyzing the light curves of active galactic nuclei (AGNs). Based on the example of AGN multiwavelength measurements, features of the following methods are considered: (1) structure functions (SFs)—to search for variability timescales; (2) discrete correlation functions (DCFs)—to search for connections between processes; (3) the Lomb–Scargle (L–S) periodogram—to search for periodicity. We analyze advantages and disadvantages of the methods and discuss their constraints and interpretation.

**Keywords:** methods: data analysis, statistical, observational; radiation mechanisms: non-thermal

**DOI:** 10.26119/VAK2024.170

# 1 Introduction

The variability in AGNs across a wide range of wavelengths is a well-studied characteristic. Its timescale ranges from few minutes through days and months to decades. The parameters of flux density variations can be used to estimate the timescales of ongoing processes, constrain the size of the emission region, and describe the evolution of flaring events across the electromagnetic spectrum. These properties help study both the underlying physical process behind the variations and possible periodicities in the light curves (LCs). We demonstrate an application of several statistical methods to the analysis of long-term multiband LCs on the example of two blazars.

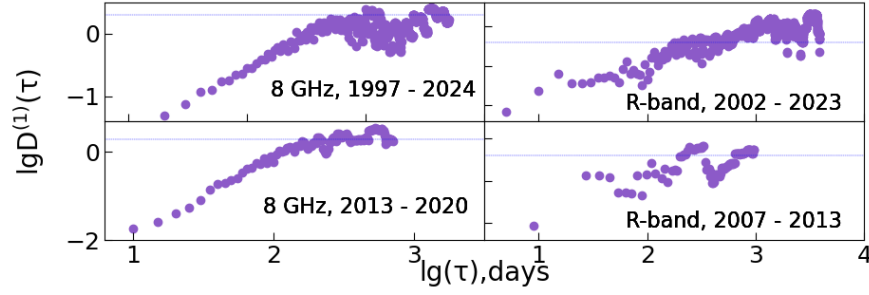
# 2 Review of the methods

**Structure function analysis.** The SF is a method of searching for typical timescales and periodicities in non-stationary processes (Simonetti et al. 1985). As it was mentioned by Emmanoulopoulos et al. (2010), results of SF analysis must be interpreted with some caution. Figure 1 shows the examples of SF analysis applied to the long-term (1997 Mar – 2024 Jul) LCs of Ton 599 at 8 GHz and of AO 0235+164 in the optical  $R$  band. SF breaks often occur in datasets without characteristic timescales. The position of these artificial breaks depends on the length of observations and the nature of the variation process. On the other side, intensive modeling shows that SFs derived from typical blazar observations resemble those coming from the shot-noise model. However, in the frequency domain the same model does not describe correctly the observations and thus may not be physically realistic. Also it was emphasised in the paper cited above that the data gaps affect severely the SF estimates in an unpredictable way, introducing systematic deviations, as it is clearly seen in Fig. 1 (bottom part on the right panel). Even the bootstrap method cannot yield statistically meaningful errors depicting the deviations between the gappy and continuous SFs. This effect as well as the presence of some powerful outbursts may be responsible for difficulties in SF estimation. Despite the drawbacks of the method, SFs have several advantages such as the ease of application, versatility, robustness to outliers, ability to detect timescales, and less sensitivity to trends.

**Discrete correlation functions.** DCFs for irregularly sampled time series are commonly used to estimate the correlation and time lags between LCs observed over approximately the same period (Edelson & Krolik 1988; Robertson et al. 2015). The method to estimate the confidence level of a resulted DCF is outlined in Emmanoulopoulos et al. (2013), and the software is provided on GitHub.<sup>5</sup> The point is

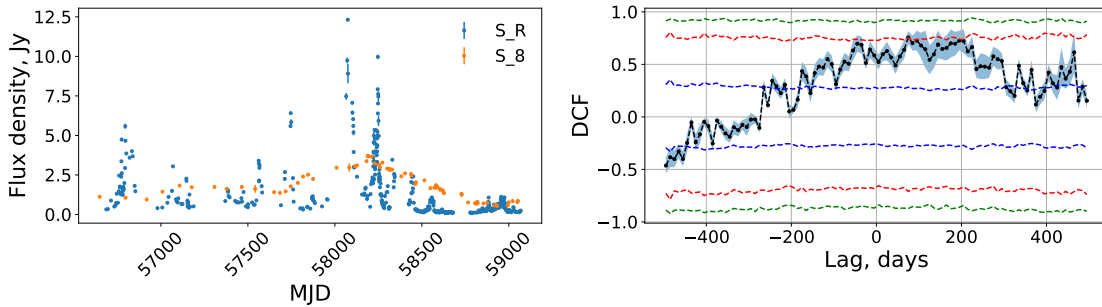
---

<sup>5</sup> <https://github.com/samconnolly/DELightcurveSimulation>

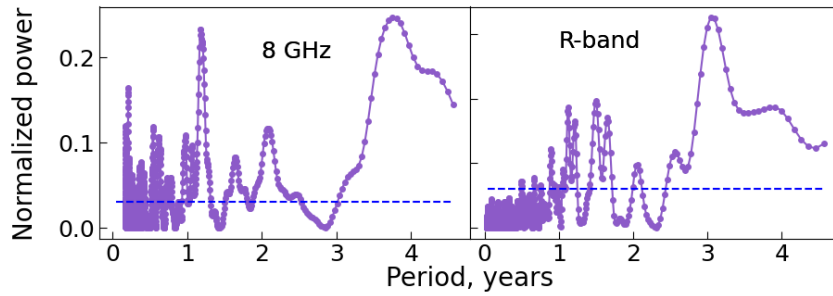


**Fig. 1.** The SFs for the total flux variations. Left: Ton 599. Right: AO 0235+164. There is no well-defined structure for the SF in the optical range (bottom left) due to systematic annual gaps in the measurements.

to simulate a large number of synthetic LCs having the same statistical properties (PDF, PSD) as the observed LC. They are simulated for all timestamps (e.g., days) and then are taken with the same window function as the observed LC has. To find the probability of getting a given DCF value purely by chance, one can calculate cross-correlation functions for an ensemble of paired artificial LCs and analyse the DCF values distribution for each time lag. An example of DCF calculation is shown in Fig. 2. Because of the discrete nature of the data, the DCF method may miss correlations at small times. Hufnagel & Bregman (1992) demonstrate that irregular time series can make it very difficult to determine the exact time delays between changes in different ranges. Also White & Peterson (1994) emphasize the importance of considering noise effects when interpreting DCF results because a high level of noise in the data leads to an increase in random correlations.



**Fig. 2.** Left: LCs of Ton 599 at 8 GHz and in the R band. Right: DCF with its confidence levels of 1, 2, and  $3\sigma$ .



**Fig. 3.** The L–S periodograms of Ton 599 for the total flux variations at 8 GHz (left) and in the optical *R* band (right). The dashed lines show the FAP = 1 per cent level (false alarm probability).

**Lomb–Scargle periodogram.** The L–S periodogram (Lomb 1976; Scargle 1982) is a standard method to search for periodicity in unevenly sampled time series. The periodogram is sensitive to time series irregularity, so its results require careful interpretation. First of all, special attention should be paid to the window function. In most cases it has a shape of a “comb” with a minimum interval of one day between the successive observations and with large gaps when the source has not been observed. The convolution of the window function with the true LC spectral image in the spectral domain can lead to false peak detection. Observational errors can increase random fluctuations in the data. Strong jitter noise can mask the peaks in the spectral domain, increasing the probability of false detections. The L–S periodograms for the radio and optical long-term LCs of Ton 599 obtained in 1997 Mar – 2024 July are shown in Fig. 3.

**Acknowledgements.** Observations with the SAO RAS telescopes are supported by the Ministry of Science and Higher Education of the Russian Federation.

## References

- Edelson R.A. and Krolik J.H., 1988, *ApJ*, 333, p. 646  
 Emmanoulopoulos D., McHardy I.M., Papadakis I.E., 2013, *MNRAS*, 433, 2, p. 443  
 Emmanoulopoulos D., McHardy I.M., Uttley P., 2010, *MNRAS*, 404, p. 931  
 Hufnagel B.R. and Bregman J.N., 1992, *ApJ*, 386, p. 473  
 Lomb N.R., 1976, *Ap&SS*, 39, 2, p. 447  
 Robertson D.R.S., Gallo L.C., Zoghbi A., et al., 2015, *MNRAS*, 453, 4, p. 3455  
 Scargle J.D., 1982, *ApJ*, 262, p. 835  
 Simonetti J.H., Cordes J.M., Heeschen D.S., 1985, *ApJ*, 296, p. 46  
 White R.J., Peterson B.M., 1994, *PASP*, 106, p. 879.