



# The influence of space curvature on pulsar braking indexes

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**Abstract.** The influence of space curvature on the input of pulsar magnetic field outside star to effective moment of inertia of neutron star and its influence on pulsar braking indexes is considered. It is shown that it leads to the changing of braking indexes estimates only in two or ten times. So a majority of pulsar braking indexes may be explained by a simple model of neutron star precessed due to its anomalous torque as well as without taking into account the space curvature influence nearby neutron star.

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# 1 Introduction

The radiopulsars are rapidly rotating neutron stars with highly strong magnetic field, surrounding neutron star (Beskin 2018). As any other matter, this magnetic field has energy and, consequently, mass. And up to light cylinder this mass rotates together with neutron star. A such mass influences the neutron star rotation (Goglichidze et al. 2015, see also Beskin & Zheltoukhov 2014), leading to star precession. This precession has period  $T_{\text{pr}} \sim 10^3 - 10^4$  years and may give input to cyclic changing of pulsar parameters and low frequency component of “red noise” (Biryukov et al. 2012). It also may be related to the repeating of FRB burst in some sources (Sob’yanin 2020). In this paper we consider the influence of space curvature on star precession and maximal possible values of pulsar braking indexes due to this precession.

# 2 Model

In the paper we use results of papers Goglichidze et al. (2015) and Matevosyan & Barsukov (2023). Following these papers let us first consider a case of magnetic field described by one harmonic with numbers  $l$  and  $m$  defined in coordinate system with axis  $Oz$  directed as  $\mathbf{e}_{lm}$ . In the paper, for the sake of simplicity, we do not consider special case  $m = \pm 1$  (Goglichidze et al. 2015). Hence, the rotational momentum  $\mathbf{L}_{lm}^f$  of such magnetic field may be written as (Goglichidze et al. 2015):

$$\mathbf{L}_{lm}^f = I_{lm}^f \boldsymbol{\Omega} + \delta I_{lm}^f \mathbf{e}_{lm} (\mathbf{e}_{lm} \cdot \boldsymbol{\Omega}), \quad (1)$$

where  $\boldsymbol{\Omega}$  is neutron star angular velocity,  $\Omega = 2\pi/P$  and  $P$  is pulsar period. Now, let us consider a neutron star with dipolar magnetic field and, in addition, a surface small scale magnetic field. We describe the dipolar magnetic field by one field harmonic  $(lm) = (10)$  and, for the sake of simplicity, we also describe a surface small scale magnetic field by one field harmonic with numbers  $(lm)$ , where  $l > 5$ . In this case the rotational momentum  $\mathbf{L}^f$  of such magnetic field may be written as (Matevosyan & Barsukov 2023):

$$\mathbf{L}^f = (I_{10}^f + I_{lm}^f) \boldsymbol{\Omega} + \delta I_{10}^f \mathbf{e}_{10} (\mathbf{e}_{10} \cdot \boldsymbol{\Omega}) + \delta I_{lm}^f \mathbf{e}_{lm} (\mathbf{e}_{lm} \cdot \boldsymbol{\Omega}). \quad (2)$$

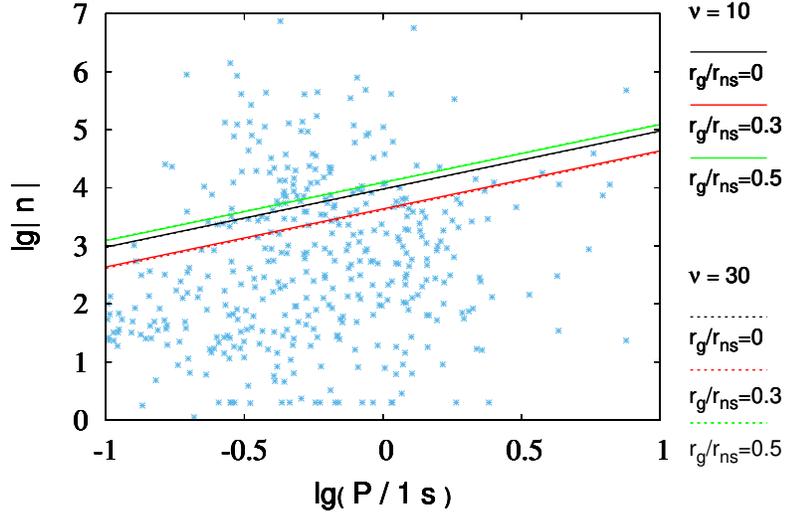
We also introduce the value  $\nu = \sqrt{\langle B_{lm}^2 \rangle / \langle B_{10}^2 \rangle}$ , where  $\langle B_{lm}^2 \rangle$  is square of magnetic induction averaged over neutron star surface (Matevosyan & Barsukov 2023). The value  $\nu$  may be considered as a ratio of harmonics on star surface. Now let us consider an axisymmetric case  $\mathbf{e}_{lm} = \mathbf{e}_{10}$ . Hence, the rotational momentum  $\mathbf{L}^f$  may be written as

$$\mathbf{L}^f = I^f \boldsymbol{\Omega} + \delta I^f \mathbf{e}_{10} (\mathbf{e}_{10} \cdot \boldsymbol{\Omega}), \quad (3)$$

where  $\delta I^f = \delta I_{10}^f + \delta I_{lm}^f$  and  $I^f = I_{10}^f + I_{lm}^f$ . This additional “star” rotational momentum leads to star precession with period  $T_{\text{pr}} \sim 10^3 - 10^4$  years. During precession the electric currents flowing through inner gaps may be modulated (Barsukov & Tsygan 2010) and, hence, we have the increasing of observed pulsar braking index  $n = \ddot{P}/(\dot{P})^2$  (Biryukov et al. 2012). The maximal input  $n_{\text{max}}$  to pulsar braking index due to such precession may be estimated as (Matevosyan & Barsukov 2023, see also Barsukov & Tsygan 2010; Biryukov et al. 2012):

$$n_{\text{max}} = \frac{K_{\text{est}}}{4\pi^2} \cdot \frac{\delta I^f}{m_{10}^2} \cdot c^3 P, \quad (4)$$

where  $K_{\text{est}} \sim 1$  is a number of the order of unity, which highly depends on exact magnetic field structure in inner gaps, pulsar inclination angle  $\chi$  and the phase of precession (Barsukov & Tsygan 2010),  $m_{10}$  is pulsar dipolar magnetic momentum, which corresponds to harmonic  $(lm) = (10)$ ,  $\chi$  is angle between vectors  $\mathbf{\Omega}$  and  $\mathbf{e}_{10}$ .



**Fig. 1.** The estimated maximal braking index values  $n_{\text{max}}$  in case of  $l = 20$  and  $m = 20$  are shown by lines. The observed braking index values  $n$  taken from Manchester et al. (2005) are shown by stars.

### 3 Results

The comparison of estimation  $n_{\max}$  of maximal possible value of braking index with observed values of braking indexes<sup>1</sup> (Manchester et al. 2005) is shown in Fig. 1. As can be seen from Fig. 1 the majority of pulsar braking indexes may be explained by this simple model. Although, some of pulsars have very large braking indexes. This could mean that its neutron star crust is highly deformed, in order to its star has a short precession period  $T_{\text{pr}}$  (Jones et al. 2017) or that we see magnetic field decay (Biryukov et al. 2017). Also, it may be related to peculiarity of braking index observation (Biryukov & Beskin 2023).

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