Fractality and kinetics of the stellar medium in the solar neighborhood based on Gaia DR2 data

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Abstract. The fractal nature of the stellar medium in the Solar vicinity has been established by studying 200 000 stars of all spectral types at distances of 1 to 100 parsec from the Sun. The fractal dimension is $D \approx 2.41$. Based on the data obtained, we examine the kinetic properties of the fractal stellar medium in the solar vicinity. We show that kinetic parameters such as correlation length, dynamic friction coefficient, effective interstellar distance for fractal stellar medium differ from the corresponding parameters for quasi-homogeneous medium with limited density fluctuations, and depend on the fractal dimension. We find that the fractal structure of the stellar medium leads to a reduction of the relaxation time in our Galaxy.

1   Introduction

The first ideas about the fractal structure of the environment of galaxies are associated with the works of Carpenter (1938) and de Vaucouleurs (1970), who showed that the spatial density in galaxy clusters satisfies the power law $n(r) \sim r^{-1.7}$, where r – the size of a cluster; n - the density of galaxies in a cluster. De Vaucouleurs extended this result not only to clusters of galaxies, but also to the entire galactic medium. He showed that the galactic medium is arranged hierarchically, and therefore any observer, included in the hierarchy, will find that the mean density around them decreases with distance. Mandelbrot (1977) proposed a fractal-hierarchical model to describe the distribution of galaxies:

$$n(r) \sim r^{-\alpha} , \tag{1}$$

where r – the characteristic size of the increasing volume around observer included in hierarchy; $n(r)$ – mean star conditional density; α – the exponent. Mandelbrot applied fractal dimension for fractal stellar medium and showed that $D = 3 - \alpha$, where D is the fractal (Hausdorff) dimension, wherein in various gravitating medium $0 \leq D \leq 3$.

Recently, we have studied fractal properties of stellar medium in the solar neighborhood from the observational data of the Gaia (DR2, 2018). Our sample included 200 000 stars of all spectral types at a distance of 1 to 100 parsec from the Sun. Our numerical calculations have shown that the mean conditional stellar density $n(r)$ is approximated by the power laws of the form:

$$n(r) = hr^{-\alpha} , \tag{2}$$

where h and α are numerical coefficients: $h = 1.654$, $\alpha = 0.586$, with the Pearson significance level $R^2 \approx 0.992$. It has confirmed the fractal-hierarchical model of de Vaucouleurs-Mandelbrot, and fractal (Hausdorff) dimension: $D = 3 - \alpha \approx 2.41$.

2   Kinetic parameters of the fractal stellar medium

In the present work we study the kinetic parameters of the fractal stellar medium in the solar neighborhood with the fractal (Hausdorff) dimension $D \approx 2.41$. In the paper Chumak & Rastorguev (2016) the important kinetic parameter, the effective interparticle (interstellar) spacing, r_{m} was obtained for the fractal stellar medium from the law of the distribution of the distance to the nearest neighbor:

$$r_{\mathrm{m}} = \frac{3}{D} \left(\frac{D}{4\pi h} \right)^{1/D} \Gamma \left(\frac{D+1}{D} \right) , \tag{3}$$

where $\Gamma(x)$ is the gamma function. Substituting our parameters for the space distribution of 200 000 stars in the solar neighborhood $h = 1.654$, $D \approx 2.41$ into the formula (3) yields an average distance between the stars $r_m \approx 0.49$ pc for the fractal stellar medium. Traditional estimations for classical model of the homogeneous stellar medium with $D = 3$ and $h \approx n \approx 0.10$ pc^{-3} (we adopt local stellar density from Binney & Tremaine 2008) yield, respectively, $r_m \approx 1$ pc, which is almost twice greater than for the fractal case. This result is due to the fact that fractal model takes into account the gravitational clustering ("clumpiness") of the stellar medium.

The second important kinetic parameter is "correlation length" r_0. Davis & Peebles (1983) for the power-law star density distribution introduced a characteristic size r_0, determined from the relation $n(r_0) = 1$. Therefore, in accordance with (2),

$$r_0 = h^{-1/(D-3)} \ . \tag{4}$$

For our fractal model of star distribution we obtain the value of the "correlation length" $r_0 = 2.35$ pc, which is approximately five times the value of the effective interparticle spacing. This value is rather theoretical and is used for different numerical estimates. In classical homogeneous stellar medium such a characteristic is absent.

Kinetic parameter, coefficient of dynamic friction a, is determined by two-point encounters. It was obtained for the fractal stellar medium in the paper Chumak & Rastorguev (2017). It can be approximately estimated using the "correlation length" r_0:

$$a_0 = a(r_0) \approx \frac{8\pi G^2 m^2 h \ln \Lambda}{\nu_0^3} r_0^{D-3} \ , \tag{5}$$

where $ln\Lambda = \ln(p_{\max}/p_{\min})$ is the Coulomb logarithm, G is the gravitational constant, $m = m_f$, the mass of a test star and the field star, $p_{\min} = 2Gm/\nu_i^2$, the impact parameter of the close encounter, in which the test star is deflected by $\pi/2$ angle, $p_{\max}$ is the cutoff parameter for distant interactions (in our case $p_{\max} = 2r_m$), v_0, the velocity of ith star at the time $t = 0$.

From formula (5) for the characteristic deceleration time of a test star as a result of dynamic friction in a fractal medium, we obtain the expression:

$$\tau_{\mathrm{relfr}} = \frac{\nu_0^3}{8\pi G^2 m^2 h \ln \Lambda} r^{3-D} \ . \tag{6}$$

For homogeneous stellar medium ($D \to 3, h \to n$) we obtain the well-known formula for the relaxation time:

$$\tau_{\mathrm{rel}} = \frac{\nu_0^3}{8\pi G^2 m^2 n \ln \Lambda} \ . \tag{7}$$

From these last two expressions we obtain the relation:

$$\tau_{\text{relfr}} = nh^{-1}r^{3-D}\tau_{\text{rel}} \ . \tag{8}$$

Substituting the above values for the solar neighborhood into this relationship, and assuming for estimates $r = r_0$, we obtain $\tau_{\text{relfr}} = 0.099\,\tau_{\text{rel}}$. Thus, the characteristic time of dynamic breaking within the fractal model is approximately ten times less than in homogeneous stellar medium.

3 Summary

Investigation of the fractal properties of the stellar medium in the solar vicinity, using the example of 200 000 stars of all spectral types at a distance of 1 to 100 pc from the Sun, indicates the presence of fractal structures with the fractal dimension $D \approx 2.41$. The study of the kinetic parameters shows that the fractal structure of the stellar medium leads to a reduction in the relaxation time in our Galaxy and that kinetic parameters such as the correlation length, dynamic friction coefficient, and effective interstellar spacing for fractal stellar medium differ significantly from the corresponding parameters for quasi-homogeneous medium.

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